Homework 4

Book Problems: Section 3.1 # 5, 13, 15 Section 3.2 # 1, 10, 11, 34 For 3.2 #1, you may use any of the methods from class to compute the determinant.

Additional Problems:

1. Let  $A = \begin{bmatrix} 2 & 2 & -2 & 1 \\ 0 & 1 & -1 & 0 \\ 6 & 0 & 0 & 7 \\ 0 & 0 & -3 & 0 \end{bmatrix}$ . Compute the determinant of A using three different methods.

- (a) The definition of the determinant
- (b) Reduction to triangular form
- (c) Cofactor expansion
- 2. Suppose A and B are  $3 \times 3$  matrices such that  $B^{-1} = \frac{1}{5}A^2$ . If det(A) = 2, what is det(B)?
- 3. Let A be a square matrix. Determine if the following are true or false. Give a proof or counterexample.
  - (a) If  $AA^T = I$ , then  $det(A) = \pm 1$ .
  - (b) If  $A + A^T = I$ , then  $det(A) = \frac{1}{2}$ .
- 4. Let A be a  $5 \times 5$  matrix with determinant 5.
  - (a) What is the RREF of A?

(b) Let 
$$\mathbf{b} = \begin{bmatrix} 5\\5\\5\\5\\5 \end{bmatrix}$$
. How many solutions are there to the linear system  $A\mathbf{x} = \mathbf{b}$ ?