Homework 3 Due: Tuesday, February 3

Book Problems:
Section 1.5 # 12, 21, 37, 50
Section 2.3 # 5, 9, 29

Additional Problems:

1. Let A and B be $n \times n$ upper triangular matrices. Determine if the following matrices are upper triangular, lower triangular, both, or neither.
   (a) $(A+B)^T$
   (b) AB
   (c) AB^T

2. Let $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
   (a) Compute D^2 and D^3.
   (b) Compute D^{100}. You do not need to simplify the entries of D^{100}.
   (c) If E is an $n \times n$ diagonal matrix with diagonal entries $e_1, e_2, ..., e_n$, what can you say about the matrix E^k for k a positive integer?

3. Let A be an $n \times n$ matrix
   (a) Prove that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.
   (b) Is the matrix $(A+A^T)(A-A^T)$ symmetric, skew symmetric, both, or neither? Either give a proof that it is always symmetric, skew symmetric, or both or find a specific example of a matrix A for which $(A + A^T)(A - A^T)$ is neither.

4. Suppose A and B are invertible 3×3 matrices and that $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 0 & 3 \\ 1 & 7 & 5 \end{bmatrix}$. Answer the following without computing A^{-1} or B.
   (a) Find $(A^{-1}B^T)^{-1}$.

(b) Let $\mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$. Find all solutions to the linear system $A^{-1}B^T\mathbf{x} = \mathbf{c}$.

5. Let $A = \begin{bmatrix} -2 & 0 & -5 & 2 \\ 1 & 0 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & 5 & 0 \end{bmatrix}$. Find A^{-1} using the methods of Section 2.3. Check your answer by computing AA^{-1} or $A^{-1}A$.