Homework 2 Due: Tuesday, January 27

Book Problems:
Section 2.2 # 1, 5, 15
Section 1.2 # 10, 11
Section 1.3 # 13, 23, 28
Section 1.4 # 3, 29

Additional Problems:
Note: Write your solutions to linear systems as vectors.

1. Solve each linear system using Gaussian elimination or Gauss-Jordan reduction.

   (a) $2x + y = 8$
   $3x + 4y = 7$
   $x + y = 3$

   (b) $x + y + z = 3$
   $2x + 2y - z = 3$

   (c) $2x + y + z = 1$
   $4x - 3y + z = 7$
   $3x - y + z = 2$

2. Let A be an $m \times n$ matrix and $\mathbf{b}$ be an m-vector such that $A\mathbf{x} = \mathbf{b}$ is a consistent linear system. Let $\mathbf{x}_p$ be a solution to $A\mathbf{x} = \mathbf{b}$ and let $\mathbf{0}$ be the zero m-vector.

   (a) Prove that if $\mathbf{x}_h$ is a solution to $A\mathbf{x} = \mathbf{0}$, then $\mathbf{x}_p + \mathbf{x}_h$ is a solution to $A\mathbf{x} = \mathbf{b}$.

   (b) Prove that if $\mathbf{x}_1$ is a solution to $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_p$ is a solution to $A\mathbf{x} = \mathbf{0}$.

   (c) Use the previous two parts to explain why the following statement is true: The solutions to $A\mathbf{x} = \mathbf{b}$ are exactly the vectors of the form $\mathbf{x}_p + \mathbf{x}_h$ where $\mathbf{x}_h$ is a solution to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

3. Let $A = \begin{bmatrix} 3 & 1 & -1 & -2 & 2 \\ 1 & 1 & 1 & 1 & 8 \\ 7 & -1 & -9 & -4 & 4 \\ 5 & 3 & 1 & -5 & -2 \end{bmatrix}$. The RREF of A is $\begin{bmatrix} 1 & 0 & -1 & 0 & 3 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Let $\mathbf{b} = \begin{bmatrix} 1 \\ -4 \\ -3 \\ 8 \end{bmatrix}$.

   (a) Find all solutions to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

(b) Prove that $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}$ is a solution to the linear system $A\mathbf{x} = \mathbf{b}$.

(c) Find all solutions to $A\mathbf{x} = \mathbf{b}$.
Hint: Use the result from problem 2c.