Homework 13 Due: Thurs, April 30

Book Problems:
Section 7.2 # 1, 11abc, 13, 19, 23

Additional Problems:

1. Let $L : P_1 \to P_1$ be the linear transformation $L(at+b) = (a+4b)t + (a+b)$. Is L diagonalizable? If yes, find a basis S for P_1 such that the representation of L with respect to S is a diagonal matrix.

2. For each matrix A, the characteristic polynomial of A is provided. Determine if A is diagonalizable.

   (a) $A = \begin{bmatrix} 4 & 4 & -2 \\ 1 & 4 & -1 \\ 3 & 6 & -1 \end{bmatrix}$, $(\lambda - 3)(\lambda - 2)^2$

   (b) $A = \begin{bmatrix} 3 & 0 & -2 & -1 \\ -1 & 2 & 5 & 4 \\ 6 & 0 & -5 & -3 \\ -6 & 0 & 4 & 2 \end{bmatrix}$, $\lambda(\lambda - 1)(\lambda + 1)(\lambda - 2)$

   (c) $A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 5 & -2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$, $(\lambda + 1)(\lambda - 1)^2(\lambda + 2)$

3. Let A be a 3×3 matrix. Suppose that the eigenvalues of A are $\lambda_1 = 1$, $\lambda_2 = 2$, and $\lambda_3 = 3$ with associated eigenvectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} -2 \\ 3 \\ -3 \end{bmatrix}$ respectively.

   Find a formula for A^k and use this to find A and A^{100} (you do not need to simplify the entries of A^{100}).