Homework 12 Due: Tues, April 28

Book Problems:
Section 7.1 # 3, 7ab, 17, 24ab, 26

Additional Problems:

1. Let $L : P_1 \to P_1$ be the linear transformation $L(at + b) = (2a + 7b)t + (2a - 3b)$. Find all eigenvalues of L. For each eigenvalue, find all associated eigenvectors.

2. For each matrix A, find the eigenvalues of A. Find a basis for the eigenspace associated with each eigenvalue.

   (a) $A = \begin{bmatrix} -1 & 0 & 0 \\ -4 & -5 & -8 \\ 4 & 4 & 7 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. Let A be an $n \times n$ matrix. Prove that A and A^T have the same eigenvalues. Do they have the same eigenvectors?

4. Let λ be an eigenvalue of an $n \times n$ matrix A with associated eigenvector $\mathbf{v}$. Prove one of the following statements (you do not need to prove all 3).

   (a) $\mathbf{v}$ is also an eigenvector of A^2 with associated eigenvalue λ^2.

   (b) $\mathbf{v}$ is also an eigenvector of A^{-1} with associated eigenvalue $1/\lambda$ (assuming A is invertible).

   (c) $\mathbf{v}$ is also an eigenvector of $A + rI$ with associated eigenvalue $\lambda + r$.