Homework 11 Due: Tues, April 21

Book Problems:
Section 6.5 # 3, 6, 9, 14, 17

Additional Problems:

1. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ y-x \end{bmatrix}$.
   Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ and $T = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \end{bmatrix} \right\}$.
   Let A be the standard matrix representing L and let B be the representation of L with respect to T. Let P be the transition matrix from T to S and let Q be the transition matrix from S to T.
   (a) Find A.
   (b) Find P and Q.
   (c) Using the methods of Section 6.5, find a formula for B in terms of A, P, Q and use this to compute B.
   (d) Compute B using the methods of Section 6.3.

2. Let $L : P_2 \to \mathbb{R}_2$ be the linear transformation $L(at^2 + bt + c) = \begin{bmatrix} a-b & b+2c \end{bmatrix}$.
   Let S, S', T, T' be the following bases for P_2 and $\mathbb{R}_2$.
   $$S = \{t^2, t, 1\}, S' = \{t^2 + 2t - 1, 3t + 5, 2t^2 + t - 4\}$$
   $$T = \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \end{bmatrix} \right\}, T' = \left\{ \begin{bmatrix} 1 & -1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \end{bmatrix} \right\}$$
   Let A be the matrix representing L with respect to S and T and let B be the matrix representing L with respect to S' and T'.
   (a) Find A.
   (b) Let C be the transition matrix from S to S' and let D be the transition matrix from T to T'. Fill in the blanks with C, C^{-1}, D, or D^{-1}:
   $$B = \_\_\_\_\_ \; A \; \_\_\_\_\_$$
   (c) Compute the transition matrices you used in the previous part and use that formula to compute B.
   (d) Compute B using the methods of Section 6.3.

3. Suppose A and B are similar matrices. Prove that $A + rI$ and $B + rI$ are similar matrices for any real number r.