Homework 10 Due: Tues, April 14

Book Problems:
Section 6.2 #1, 5, 6, 19, 25
Section 6.3 # 1, 10, 13

Additional Problems:

1. Let $L : \mathbb{R}^2 \to P_1$ be the linear transformation $L\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = (2a+5b)t + (a+3b)$. Show that L is invertible and find L^{-1}.

2. Let $L : P_2 \to \mathbb{R}^4$ be the linear transformation $L(at^2+bt+c) = \begin{bmatrix} a+b+c \\ a-b+c \\ 2b \\ b-a-c \end{bmatrix}$.

   (a) Find a basis for $\ker L$.

   (b) Find a basis for range$L$.

   (c) Find the representation of L with respect to S and T where S and T are the following bases for P_2 and $\mathbb{R}^4$ respectively. $S = \{t^2+2t-1, 3t+5, 2t^2+t-4\}$, $T = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. Let $L : M_{nn} \to \mathbb{R}$ be the linear transformation $L(A) = a_{11} + a_{22} + ... + a_{nn}$ where a_{ij} is the i,j-th entry of A. Find $\dim\ker L$ and $\dim \text{range}L$ (your answers may depend on n). Is L one-to-one? Onto?

4. Let $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ and let $T = \left\{ \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$. These are bases for $\mathbb{R}^3$ and M_{22} respectively. Let $L : \mathbb{R}^3 \to M_{22}$ be a linear transformation such that the representation of L with respect to S and T is $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \\ 1 & 4 & -1 \end{bmatrix}$. Find $L\left(\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}\right)$.