Homework 1 Due: Tuesday, January 20

Book Problems:
Section 1.1 # 3, 7, 17
Section 2.1 # 1, 3, 9

Additional Problems:

1. Let c be a constant. Consider the linear system:

$$x + y + z = -1$$

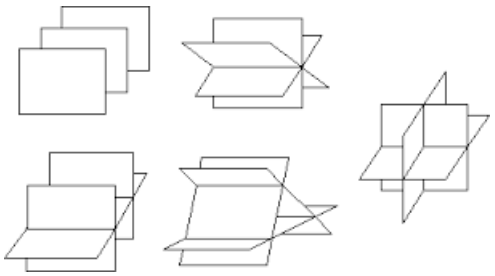
$$x - y + 3z = c^2$$

$$x + 2z = 4$$

   For what value or values of c is the system consistent?

2. Describe the possible numbers of solutions to a linear system of 3 equations in 3 unknowns.

   Hint: The solutions to a linear system of 3 equations in 3 unknowns are the intersection points of 3 planes in 3-dimensional space. The following picture demonstrates the ways in which 3 different planes can intersect. You should also consider what happens if 2 or 3 of the equations are the same plane.



3. Consider the following homogeneous linear system.

$$2x + y - z + w = 0$$

$$-x - 2y + z - 2w = 0$$

   (a) Show that $x_1 = 1, y_1 = -2, z_1 = 3, w_1 = 3$ is a solution to the linear system and that $x_2 = 0, y_2 = 1, z_2 = 0, w_2 = -1$ is also a solution to the linear system.

   (b) Is $x_3 = x_1 + x_2, y_3 = y_1 + y_2, z_3 = z_1 + z_2, w_3 = w_1 + w_2$ a solution to the linear system?

(c) Is $x_4 = 5x_1, y_4 = 5y_1, z_4 = 5z_1, w_4 = 5w_1$ a solution to the linear system?

4. (a) Find the coefficient and augmented matrices of the following linear system.

$$x + 3y - z + w = 4$$

$$2x + 2w = 1$$

$$z - 6w = 0$$

(b) Find the linear system whose augmented matrix is $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 3 & 3 & 4 & -2 \end{array}\right]$.

5. Find the reduced row echelon form of the following matrix: $\begin{bmatrix} 0 & 1 & 2 & 5 \\ 3 & -6 & 9 & 3 \\ 2 & -3 & 8 & 9 \end{bmatrix}$

Write down the row operations you perform using the notation for elementary row operations.