Math 3333
Spring 2015
Midterm 3

Name:______________________________

Problem	Points
Problem 1 (10pts)	
Problem 2 (12pts)	
Problem 3 (10pts)	
Problem 4 (10pts)	
Problem 5 (14pts)	
Problem 6 (14pts)	
Problem 7 (30pts)	
Total	

1. Let S be the following orthonormal basis for $\mathbb{R}^3$.

$$S = \left\{ \frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}, \frac{1}{3}\begin{bmatrix}-2\\2\\-1\end{bmatrix}, \frac{1}{3}\begin{bmatrix}2\\1\\-2\end{bmatrix} \right\}$$

If $\mathbf{v} = \begin{bmatrix}2\\1\\4\end{bmatrix}$, find $[\mathbf{v}]_S$. (10 pts)

2. Let V be a 2-dimensional subspace of $\mathbb{R}^n$ with basis $T = \{\mathbf{v_1}, \mathbf{v_2}\}$.
Suppose that $\|\mathbf{v_1}\| = \sqrt{3}$, $\|\mathbf{v_2}\| = \sqrt{5}$, and $\mathbf{v_1} \cdot \mathbf{v_2} = 2$.
Let $\mathbf{u} = \mathbf{v_1} - 2\mathbf{v_2}$. Find $\|\mathbf{u}\|$. (12 pts)

3. Let $L : \mathbb{R}^2 \to \mathbb{R}$ be the function $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \sqrt{x^2 + y^2}$.

   Is L a linear transformation? Why or why not? (10 pts)

4. Let V be a 3-dimensional space. Let $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ and $T = \{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ be bases for V with transition matrix from S to T equal to $Q = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & -4 \end{bmatrix}$.

   Let $\mathbf{u} = 2\mathbf{v_1} + 3\mathbf{v_2} + \mathbf{v_3}$. Find real numbers a, b, c such that $\mathbf{u} = a\mathbf{w_1} + b\mathbf{w_2} + c\mathbf{w_3}$. (10 pts)

5. Let V be the 3-dimensional subspace of $\mathbb{R}^4$ with basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ -3 \end{bmatrix} \right\}$.

   Find an orthonormal basis for V. (14 pts)

6. Let $L : P_3 \to \mathbb{R}^4$ be a linear transformation with $\dim \operatorname{range} L = 4$.

(a) Prove that L is invertible. (6 pts)

(b) Suppose that $L(t^3) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ and $L(t) = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$. Find $L^{-1}\left(\begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}\right)$. (8 pts)

7. Let $L : M_{22} \to \mathbb{R}^2$ be the linear transformation $L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ c-d \end{bmatrix}$.

(a) Find a basis for the kernel of L. (8 pts)

(b) Find a basis for the range of L. (6 pts)

(c) Is L one-to-one? Is L Onto? (4 pts)

(d) Find the representation of L with respect to S and T where
$S = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \right\}$ and $T = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.
(12 pts)