

Math 3333

Spring 2015

Midterm 1

Name: _____

Problem	Points
Problem 1 (8pts)	
Problem 2 (12pts)	
Problem 3 (24pts)	
Problem 4 (24pts)	
Problem 5 (12pts)	
Problem 6 (22pts)	
Total	

1. Is the vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$?
Why or why not? (8 pts)

2. Suppose A and B are $n \times n$ matrices and that A is symmetric and B is skew symmetric. Determine if AB is symmetric, skew symmetric, both, or neither.

Either give a proof (if your answer is symmetric, skew symmetric, or both), or find an example which shows that AB can be neither. (12 pts)

3. Let $A = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 5 & 0 & -1 \\ 2 & 0 & 3 & 4 \\ 0 & -1 & 1 & 0 \end{bmatrix}$. Compute the following determinants. (24 pts)

(a) $\det(A)$

(b) $\det(2A^{-1})$

(c) $\det(A^T A)$

(d) $\det(A^T - A)$

4. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 6 \\ 1 & 1 & 4 \end{bmatrix}$.

(a) Find A^{-1} or show that A is not invertible. (12 pts)

(b) Use your answer to part (a) to find the solutions to $A^2\mathbf{x} = \mathbf{b}$ where

$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. (12 pts)

5. Suppose A is a 4×3 matrix and that the linear system $A\mathbf{x} = \mathbf{b}$ has exactly one solution for some 4-vector \mathbf{b} .
- (a) If possible, find the RREF of A . Otherwise describe what can be said about the RREF of A from the given information. (6 pts)
- (b) If \mathbf{c} is another 4-vector, what are the possible numbers of solutions to the linear system $A\mathbf{x} = \mathbf{c}$? (6 pts)

6. Consider the linear system:

$$-a - 2b + d - 2e = 2$$

$$a + 2b + 3c - 5d + 15e = -11$$

$$2a + 4b + c - d + 6e = 7$$

(a) Find the augmented matrix of the linear system. (6 pts)

(b) One of the following is the reduced row echelon form (RREF) of the augmented matrix. Circle it. (8 pts)

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 1 & 4 \\ 0 & 1 & 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 1 & -1 & 6 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & -2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 1 & -1 & 6 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 3 & -11 \\ 0 & 0 & 0 & 1 & -1 & 7 \end{array} \right]$$

(c) Find all solutions to the linear system. Write your answer as a vector. (8 pts)