Quiz 3 Solutions

1. Determine if the following set is orthogonal, orthonormal, or neither, where the inner product is the dot product on $\mathbb{R}^3$. (3 pts)

$$\left\{ \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \right\}$$

   This set is neither. The dot product of the first and third vectors is $-1/\sqrt{3}$ so it is not orthogonal. Note that these vectors are all length 1, but the set is not orthonormal because it is not orthogonal.

2. Let V be an inner product space and let $\mathbf{v_1}, \mathbf{v_2}$ be vectors in V. Suppose that $(\mathbf{v_1}, \mathbf{v_1}) = 4, (\mathbf{v_1}, \mathbf{v_2}) = -2$, and $(\mathbf{v_2}, \mathbf{v_2}) = 1$. Let $\mathbf{w} = \mathbf{v_1} - 3\mathbf{v_2}$. Compute $\|\mathbf{w}\|$. (5 pts)
   $\|\mathbf{w}\| = \sqrt{(\mathbf{w}, \mathbf{w})}$ so we first compute the inner product of $\mathbf{w}$ with itself. Using the properties of inner products, we get that $(\mathbf{w}, \mathbf{w}) = (\mathbf{v_1} - 3\mathbf{v_2}, \mathbf{v_1} - 3\mathbf{v_2}) = (\mathbf{v_1}, \mathbf{v_1}) - 3(\mathbf{v_1}, \mathbf{v_2}) - 3(\mathbf{v_2}, \mathbf{v_1}) + (-3)^2(\mathbf{v_2}, \mathbf{v_2}) = (\mathbf{v_1}, \mathbf{v_1}) - 6(\mathbf{v_1}, \mathbf{v_2}) + 9(\mathbf{v_2}, \mathbf{v_2}) = 4 - 6(-2) + 9(1) = 25$. Then $\|\mathbf{w}\| = \sqrt{25} = 5$.

3. If W is a 1-dimensional subspace of $\mathbb{R}^4$, what is the dimension of $W^\perp$? (2 pts)

   Using the formula $\dim V = \dim W + \dim W^\perp$ we get that $4 = 1 + \dim W^\perp$ so $\dim W^\perp = 3$.