1. Find the reduced row echelon form of the following matrix. Make sure to specify the row operations that you use. (4 pts)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} r_1 \leftrightarrow r_2 \begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \tfrac{1}{2}r_3 \to r_3 \begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} r_2 - r_3 \to r_2$$
$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} r_1 + r_2 \to r_1 \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Find all solutions to each of the following linear systems. Write your answer as a vector. (2 pts each)

(a) $\left[\begin{array}{ccc:c} 1 & 2 & 0 & -3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

This is the system $x + 2y = -3, y - z = 2, z = 3$. Using backtracking we get $z = 3, y = 5, x = -13$ so the only solution is $\begin{bmatrix} -13 \\ 5 \\ 3 \end{bmatrix}$.

(b) $\left[\begin{array}{ccc:c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

This system has no solutions, the last equation is $0 = 1$.

(c) $\left[\begin{array}{cccc:c} 1 & 4 & 0 & -2 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Using variables x, y, z, w, there are no leading ones in columns 2 or 4 so the variables y and w can be anything and the others can be solved for in terms of y, w. The system is $x + 4y - 2w = 1$, $z = 3$ so we get that $z = 3$ and $x = 1 - 4y + 2w$. The solutions are all vectors of the form $\begin{bmatrix} 1 - 4y + 2w \\ y \\ 3 \\ w \end{bmatrix}$ where y, w can be anything.