Homework 9 Due: Friday, October 17

1. Find a basis for the subspace of P_3 spanned by
$$\{t^3+t, t-1, t^3+1, t^2+t, t^3+t^2+t+1\}\ .$$

2. Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \\ 4 & 1 & 1 & 2 \\ -1 & 3 & -3 & 7 \end{bmatrix}$.

   (a) Find a basis for the row space of A which consists of vectors which are row vectors of A.

   (b) Find another basis for the row space of A which consists of vectors that are not row vectors of A.

   (c) Is the vector $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$ in the row space of A?

3. Let $A = \begin{bmatrix} 1 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & -1 & 3 \\ 2 & 2 & 7 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$.

   (a) Find the rank and nullity of A.

   (b) Find a basis for the row space of A.

   (c) Find a basis for the column space of A.

   (d) Find a basis for the null space of A.

4. Let A be a 4×6 matrix with rank 4.

   (a) Find the nullity of A.

   (b) How many solutions does $A\mathbf{x} = \mathbf{0}$ have?

   (c) Find the dimension of the row space of A.

   (d) Are the rows of A linearly independent?

   (e) Find the dimension of the column space of A.

   (f) Are the columns of A linearly independent?

5. Let A be a 5×3 matrix with rank 3. Find the RREF of A.

6. Let A be a 3×4 matrix. Write $\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}$ for the rows of A. Suppose we know the following: $\mathbf{r_3} \neq \mathbf{0}$, $\mathbf{r_2}$ is not a multiple of $\mathbf{r_3}$, and $\mathbf{r_1}$ is contained in span$\{\mathbf{r_2}, \mathbf{r_3}\}$. Find the rank and nullity of A.

7. Let A be an $m \times n$ matrix with $m \neq n$. Show that either the rows of A are linearly dependent or the columns of A are linearly dependent.