1. Find a basis for and the dimension of the null space of $A = \begin{bmatrix} 2 & -2 & 1 & 7 \\ 1 & -1 & 1 & 4 \\ -3 & 3 & 8 & -1 \\ -2 & 2 & 8 & 2 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 1 & -3 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

   (a) Find all real numbers λ such that $(\lambda I - A)\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

   (b) For each λ from part (a), find a basis for the solution space of $(\lambda I - A)\mathbf{x} = \mathbf{0}$.

3. Let A be a 3×3 matrix. Find all possible values for the nullity of A if:

   (a) $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

   (b) The RREF of A has 2 nonzero rows and one zero row.

   (c) A is row equivalent to a matrix B with nullity 2.

   (d) A is not the zero matrix.

   (e) $\det(A) \neq 0$

4. The matrix $A = \begin{bmatrix} -1 & -2 & -5 & -11 & 5 \\ 3 & 4 & 13 & 29 & -4 \\ 2 & -2 & 4 & 10 & 6 \\ 1 & 1 & 4 & 9 & -2 \end{bmatrix}$ has RREF $\begin{bmatrix} 1 & 0 & 3 & 7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) Find all solutions to $A\mathbf{x} = \mathbf{0}$. Write your answer as a vector.

   (b) Let $\mathbf{b} = \begin{bmatrix} 7 \\ -10 \\ 2 \\ -4 \end{bmatrix}$. Show that $\begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ is a solution to $A\mathbf{x} = \mathbf{b}$.

   (c) Use your answers to the previous 2 parts to find all solutions to $A\mathbf{x} = \mathbf{b}$. Write your answer as a vector.

5. Let $S = \{t^2 - 1, t + 2, 3t\}$ be an ordered basis for P_2.

   (a) Find the coordinate vector of $p(t) = 3t^2 + t + 1$ with respect to S.

   (b) Find $q(t)$ where $q(t)$ has coordinate vector $\begin{bmatrix} 7 \\ -1 \\ 1 \end{bmatrix}$.

6. Let $S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ and $T = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}$ be ordered bases for $\mathbb{R}^2$. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

   (a) Find the coordinate vectors $[\mathbf{v}]_S$ and $[\mathbf{v}]_T$.

   (b) Find the transition matrix $P_{S \leftarrow T}$ from T to S.

   (c) Find the transition matrix $Q_{T \leftarrow S}$ from S to T.

   (d) Verify that $[\mathbf{v}]_S = P_{S \leftarrow T}[\mathbf{v}]_T$ and $[\mathbf{v}]_T = Q_{T \leftarrow S}[\mathbf{v}]_S$.

   (e) How are $P_{S \leftarrow T}$ and $Q_{T \leftarrow S}$ related?

7. Let $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ be an ordered basis for $\mathbb{R}^3$. Suppose T is an ordered basis for $\mathbb{R}^3$ such that $P_{S \leftarrow T} = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ and $\mathbf{v}$ is a vector in $\mathbb{R}^3$ with $[\mathbf{v}]_T = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$.

   (a) Find the coordinate vector $[\mathbf{v}]_S$.

   (b) Find the basis T.

   (c) Find $\mathbf{v}$.