Homework 3 Due: Monday, September 8

Book Problems: Section 1.5 # 14, 40, 43
Section 2.3 # 5, 9, 12a, 20

Additional Problems:

1. (a) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$. Compute AB.

   (b) Let C be an $n \times n$ diagonal matrix with diagonal entries $c_1, c_2, ..., c_n$ and D be an $n \times n$ diagonal matrix with diagonal entries $d_1, d_2, ..., d_n$. Describe the matrix CD.

   (c) Determine if the following statement is true or false. Either explain why it is always true, or present a counterexample to show it is false.
   If C and D are diagonal $n \times n$ matrices then $CD = DC$.

2. (a) Write down an example of a 3×3 upper triangular matrix which is not diagonal and a 3×3 lower triangular matrix which is not diagonal.

   (b) Determine if the following statement is true or false. Either explain why it is always true, or present a counterexample to show it is false.
   If A is an upper triangular $n \times n$ matrix and B is a lower triangular $n \times n$ matrix then AB is a diagonal matrix.

3. Determine if each matrix is symmetric, skew symmetric, both, or neither.

   (a) AA^T where A is an $n \times m$ matrix

   (b) $A + A^T$ where A is an $n \times n$ matrix

   (c) $A - A^T$ where A is an $n \times n$ matrix

   (d) AB where A and B are $n \times n$ symmetric matrices

   (e) A^3 where A is an $n \times n$ skew symmetric matrix

4. Suppose A and B are $n \times n$ matrices such that $AB = 0$. Prove the following statements about A and B.

   (a) If A is invertible then $B = 0$.

   (b) If $B \neq 0$ then A is not invertible.