Homework 14 Due: Monday, December 1

1. Let $L : P_1 \to P_1$ be the linear operator $L(at + b) = (b - a)t + 5b$. Find all eigenvalues of L and all associated eigenvectors.

2. Let $A = \begin{bmatrix} 1 & 5 & 2 & -5 & c \\ 1 & 2 & 3 & 4 & 0 \\ 3 & -6 & 11 & 1 & 1 \\ 2 & 2 & 2 & 1 & 3 \\ 3 & 9 & 6 & -8 & 0 \end{bmatrix}$. For what value or values of c is $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ an eigenvector of A? What is the associated eigenvalue?

3. Find the eigenvalues of A. For each eigenvalue, find a basis for the associated eigenspace.

   (a) $A = \begin{bmatrix} 0 & -3 & -1 \\ -1 & 2 & 1 \\ 3 & -9 & -4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. Suppose A is an invertible $n \times n$ matrix and λ is an eigenvalue of A. Prove that $\lambda \neq 0$ and $1/\lambda$ is an eigenvalue of A^{-1}.

5. Let A be a matrix with eigenvalues $\lambda_1 \neq \lambda_2$. Let W_1 be the eigenspace associated with λ_1 and W_2 be the eigenspace associated with λ_2. Prove that $W_1 \cap W_2 = \{\mathbf{0}\}$ (i.e. that the only vector in both eigenspaces is the zero vector).

6. Determine if the following statements are true or false. Give a proof or a counterexample.

   (a) If -5 is an eigenvalue of A, then 25 is an eigenvalue of A^2.

   (b) If A and B are similar matrices and $\mathbf{x}$ is an eigenvector of A, then $\mathbf{x}$ is an eigenvector of B.