Homework 11 Due: Monday, November 3

For all problems involving $\mathbb{R}^n$, you may assume the inner product is the dot product unless otherwise specified.

1. Let $S = \left\{ \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}, \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix} \right\}$.

   Verify that S is an orthonormal basis for $\mathbb{R}^4$. Let $\mathbf{v} = \begin{bmatrix} 4 \\ -1 \\ 2 \\ 7 \end{bmatrix}$. Use dot products to find $[\mathbf{v}]_S$.

2. Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$. S is a basis for $\mathbb{R}^3$. Use the Gram-Schmidt process to transform S into:

   (a) an orthogonal basis.

   (b) an orthonormal basis.

3. Use the Gram-Schmidt process to find an orthonormal basis for W where W is the subspace of $\mathbb{R}^4$ which consists of all vectors of the form $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ such that $a + b + c + d = 0$.

4. Let V be P_2 with inner product $(p(t), q(t)) = \int_0^1 p(t)q(t)\ dt$. Use the Gram-Schmidt process to transform the basis $\{1, t, t^2\}$ into an orthogonal basis.

5. Let W be the subspace of $\mathbb{R}^3$ spanned by $\left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} \right\}$.

   (a) Find a basis for $W^\perp$.

   (b) Find $\dim W$ and $\dim W^\perp$.

   (c) Describe W and $W^\perp$ geometrically.

6. Let $A = \begin{bmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 2 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 0 & -3 \end{bmatrix}$. Let U be the null space of A and W be the column space of A^T. Note that U and W are both subspaces of $\mathbb{R}^5$.

   (a) Find a basis for U.

   (b) Find a basis for W.

   (c) Show that if $\mathbf{u}$ is in U and $\mathbf{w}$ is in W then $\mathbf{u} \cdot \mathbf{w} = 0$.

7. Let V be a finite dimensional inner product space and let W be a subspace of V. Find $\dim W^{\perp}$ if:

   (a) $\dim V = 7$ and $\dim W = 3$.

   (b) $V = \mathbb{R}^2$ and W is a line through the origin.

   (c) $V = \mathbb{R}^3$ and W is a line through the origin.