

1. Find all values of c for which the linear system:

$$x + y + z = 1$$

$$x - y = 3$$

$$x + y + z = c^2$$

$$2x + cz = 4$$

- (a) has no solutions.
 (b) has exactly one solution.
 (c) has infinitely many solutions.
 (d) is consistent.
2. Find the augmented matrix of the linear system.

$$x + 3y = z + 5$$

$$z - y + x = 7$$

$$4y + 5x = 0$$

$$x + 7z + 1 = 3$$

3. Find the homogeneous linear system with coefficient matrix $A = \begin{bmatrix} 1 & 0 & 2 & 6 & 0 \\ 0 & 1 & 3 & 0 & 9 \\ 4 & 0 & 0 & 1 & -1 \end{bmatrix}$.

4. Determine if each matrix is in row echelon form (REF), reduced row echelon form (RREF), or neither.

(a) $\begin{bmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 7 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

5. For what point or points (a, b, c) is the following matrix in row echelon form? reduced row echelon form?

$$\begin{bmatrix} 1 & a & 0 \\ 0 & b & 1 \\ 0 & 0 & c \end{bmatrix}$$

6. What are the possibilities for the number of solutions to a linear system? Explain how you can determine the number of solutions from the REF or RREF of the augmented matrix of the system.
7. The following augmented matrices are in REF or RREF. Determine the number of solutions to the corresponding linear system. Find all solutions - your answer should be written as a vector.

$$(a) \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 7 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

$$(b) \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$(c) \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 1 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

8. Find the augmented matrix of each linear system and put it into RREF. Make sure to write down the row operations that you are doing. Use the RREF to find all solutions to the linear systems.

$$(a) \begin{aligned} x + 2y - 3z &= -5 \\ 3x + 6y - 8z &= -13 \\ -x - 2y + 2z &= 3 \end{aligned}$$

$$(b) \begin{aligned} 3x + 4y - 2z &= 2 \\ 2y &= 4 \\ x + y + z &= 0 \end{aligned}$$

9. Suppose A is a 4×4 matrix for which the homogeneous linear system with coefficient matrix A has exactly one solution. Find the RREF of A .
10. Determine if each statement is true or false.
- If A and B are matrices with the same RREF, then A and B are row equivalent to each other.
 - Any two linear systems whose coefficient matrices are row equivalent have the same solutions.
 - Given a homogeneous linear system of m equations and n unknowns, if $m < n$ then the system has infinitely many solutions.
 - Given a homogeneous linear system of m equations and n unknowns, if $n < m$ then the system has exactly one solution.