Math 3333
Fall 2014
Final Exam

Name:____

Problem	**Points**
Problem 1 (15pts)	
Problem 2 (12pts)	
Problem 3 (18pts)	
Problem 4 (18pts)	
Problem 5 (13pts)	
Problem 6 (24pts)	
Bonus (5pts)	
Total	

1. Which of the following sets are subspaces of $\mathbb{R}^3$? Circle yes if it is a subspace and no if it is not. (15 pts)

   (a) A plane through the origin in $\mathbb{R}^3$. yes/no

   (b) A line in $\mathbb{R}^3$ which does not go through the origin. yes/no

   (c) The origin. yes/no

   (d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} \right\}$ yes/no

   (e) A sphere of radius 1 in $\mathbb{R}^3$ centered at the origin. yes/no

   (f) A ball of radius 1 in $\mathbb{R}^3$ centered at the origin (this is the sphere and its interior) yes/no

   (g) The solutions to the linear system $A\mathbf{x} = \mathbf{b}$ where A is a fixed 3×3 matrix and $\mathbf{b}$ is a fixed nonzero vector. yes/no

   (h) The set of all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $z = xy$. yes/no

   (i) The column space of a 3×5 matrix. yes/no

   (j) The null space of a 4×3 matrix. yes/no

2. Let W be the subspace of $\mathbb{R}^4$ which consists of vectors $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ such that $a + c = b + d$.

(a) Find a basis for W and the dimension of W. (6 pts)

(b) Assuming the dot product on $\mathbb{R}^4$, find a basis for $W^{\perp}$. (6 pts)

3. Let $A = \begin{bmatrix} -4 & -7 & 7 \\ -1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

(a) Find the eigenvalues of A. (6 pts)

(b) For each eigenvalue, find a basis for the associated eigenspace. (8 pts)

(c) Is A diagonalizable? Why or why not? (4 pts)

4. Let $L : P_1 \to \mathbb{R}^3$ be the linear transformation $L(at+b) = \begin{bmatrix} a+2b \\ a+b \\ a-b \end{bmatrix}$.

(a) Find the dimension of the kernel L and the dimension of the range of L. (6 pts)

(b) Is L one-to-one? Is L onto? (4 pts)

(c) Find the representation of L with respect to S and T where

$S = \{5t-1, 2t+3\}$ and $T = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. (8 pts)

5. Let $V = \mathbb{R}^3$ with the following inner product:

$$\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} d \\ e \\ f \end{bmatrix} \right) = (a-b)(d-e) + 4be + cf$$

(a) Find the length of the vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. (5 pts)

(b) Determine if the set $S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} \right\}$ is orthogonal, orthonormal, or neither. (8 pts)

6. Let $A = \begin{bmatrix} -2 & 0 & 0 \\ -4 & 7 & -4 \\ -1 & 6 & -3 \end{bmatrix}$. Let $S = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \right\}$.

(a) Prove that the vectors in S are eigenvectors of A and find their associated eigenvalues. (6 pts)

(b) Find a diagonal matrix D and an invertible matrix P such that $D = P^{-1}AP$. (6 pts)

(c) Find the inverse of the matrix P from part (b). (6 pts)

(d) Find A^{50}. (6 pts)
Note: Your answer should be a single matrix. The entries of the matrix do not need to be simplified (they can contain terms like r^{50}).

Bonus: Find the determinant of the following 16×16 matrix. (5 pts)
You must show work to get credit.

$$\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}$$