Math 3333
Fall 2014
Midterm 3

Name:____

Problem	**Points**
Problem 1 (20pts)	
Problem 2 (18pts)	
Problem 3 (10pts)	
Problem 4 (26pts)	
Problem 5 (26pts)	
Total	

1. Let V be an inner product space and let $\mathbf{u}$ and $\mathbf{v}$ be vectors in V. Suppose that $\|\mathbf{u}\| = \sqrt{3}$, $\|\mathbf{v}\| = 4$ and the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{6}$. Compute the following inner products.
The following may be useful: $\sin(\frac{\pi}{6}) = \frac{1}{2}$ and $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

(a) $(\mathbf{u}, \mathbf{u})$ and $(\mathbf{v}, \mathbf{v})$ (6 pts)

(b) $(\mathbf{u}, \mathbf{v})$ (6 pts)

(c) $(\mathbf{u} + \mathbf{v}, 2\mathbf{u} - \mathbf{v})$ (8 pts)

2. Let W be a subspace of the inner product space $\mathbb{R}^4$ with the dot product.

Suppose W has basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 4 \\ 0 \end{bmatrix} \right\}$.

(a) Find an orthonormal basis for W. (12 pts)

(b) Is the vector $\begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ in $W^{\perp}$? Why or why not? (6 pts)

3. Let $L : M_{nn} \to M_{nn}$ be the function $L(A) = A^T A$. Is L a linear transformation? Why or why not? (10 pts)

4. Let $L : P_3 \to \mathbb{R}^3$ be the linear transformation defined by

$$L(at^3 + bt^2 + ct + d) = \begin{bmatrix} a - b + c \\ d + 2b - 2c \\ b - c \end{bmatrix} .$$

(a) Find a basis for the kernel of L. (8 pts)

(b) Find the dimension of the range of L. Is L onto? (8 pts)

(c) Find the representation of L with respect to S and T where
$S = \{1, t, t^2, t^3\}$ and $T = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ (10 pts)

5. Let $L : V \to V$ be a linear transformation. Let $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be a basis for V. Suppose we know the following:

$$L(\mathbf{v_1}) = \mathbf{v_1} + \mathbf{v_3}$$

$$L(\mathbf{v_2}) = \mathbf{v_1} + 2\mathbf{v_2} + 3\mathbf{v_3}$$

$$L(\mathbf{v_3}) = 2\mathbf{v_3}$$

(a) Find $L(2\mathbf{v_1} - \mathbf{v_2})$. (6 pts)

(b) Find the representation of L with respect to S. (8 pts)

(c) Prove that L is invertible and find $L^{-1}(\mathbf{v_3})$. (12 pts)