# Math 3333
# Fall 2014
# Midterm 2

Name:________________________________________

| **Problem** | **Points** |
|---|---|
| Problem 1 (10pts) | |
| Problem 2 (10pts) | |
| Problem 3 (10pts) | |
| Problem 4 (20pts) | |
| Problem 5 (28pts) | |
| Problem 6 (22pts) | |
| Total | |

1. Let $V$ be the set of all real numbers with operations $\mathbf{u} \oplus \mathbf{v} = \mathbf{u} + \mathbf{v}$ and $c \odot \mathbf{u} = |c|\mathbf{u}$ (where $|c|$ is the absolute value of $c$). Prove that $V$ with the operations $\oplus$ and $\odot$ is NOT a vector space by finding a property from the definition of a vector space which is not satisfied. (10 pts)

2. Let $W$ be the set of all $2 \times 2$ matrices with determinant 0. Is $W$ a subspace of $M_{22}$? Why or why not? (10 pts)

3. Let $W$ be the subspace of $P_3$ which consists of all polynomials of the form $p(t) = at^3 + bt^2 + ct + d$ with $a + d = 2b$. Find a basis for $W$ and $\dim W$. (10 pts)

4. Let $S = \{ \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 2 & -2 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 0 & 0 & 0 \end{bmatrix} \}$.

(a) Find a basis for span $S$. What is the dimension of span $S$? (12 pts)

(b) Circle yes or no. You do not need to explain your answer. (2 pts each)

Does $S$ span $\mathbb{R}_4$? yes/no

Is $S$ linearly independent? yes/no

Does $S$ contain a basis for $\mathbb{R}_4$? yes/no

Is $S$ contained in a basis for $\mathbb{R}_4$? yes/no

5. Let $V$ be a 2-dimensional space with basis $S = \{\mathbf{v_1}, \mathbf{v_2}\}$. Let $T = \{\mathbf{w_1}, \mathbf{w_2}\}$ where $\mathbf{w_1} = \mathbf{v_1} - \mathbf{v_2}$ and $\mathbf{w_2} = 2\mathbf{v_1} + 3\mathbf{v_2}$.

   (a) Show that $T$ is also a basis for $V$. (12 pts)

   (b) Find the transition matrix $P_{S \leftarrow T}$ from $T$ to $S$. (8 pts)

   (c) If $\mathbf{v}$ is a vector in $V$ with $[\mathbf{v}]_T = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, what is $[\mathbf{v}]_S$? (8 pts)

6. Let $A = \begin{bmatrix} 3 & 6 & 2 & -1 & -4 \\ 1 & 2 & 3 & 0 & -2 \\ -2 & -4 & 1 & 1 & 2 \\ 5 & 10 & 1 & -2 & -6 \\ 3 & 6 & 6 & 1 & 2 \\ 2 & 4 & -5 & -3 & -8 \end{bmatrix}$. The RREF of $A$ is $\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find the rank and nullity of $A$. (4 pts)

(b) Find a basis for the column space of $A$. (5 pts)

(c) Find a basis for the row space of $A$. (5 pts)

(d) Find a basis for the null space of $A$. (8 pts)