Math 3333
Fall 2014
Midterm 1

Name:____

Problem	**Points**
Problem 1 (12pts)	
Problem 2 (8pts)	
Problem 3 (12pts)	
Problem 4 (18pts)	
Problem 5 (17pts)	
Problem 6 (16pts)	
Problem 7 (17pts)	
Total	

1. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   Circle yes or no to the following questions. You do not need to show work on this problem. (12 pts)

   (a) Is A a diagonal matrix? Yes/No

   (b) Is A a scalar matrix? Yes/No

   (c) Is A an upper triangular matrix? Yes/No

   (d) Is A a symmetric matrix? Yes/No

   (e) Is A an invertible matrix? Yes/No

   (f) Is A in row echelon form? Yes/No

2. The following 2 questions are multiple choice. Circle one answer for each question. You do not need to show work on this problem. (8 pts)

   (a) Suppose A is a 5×5 matrix and $\det(A) = 3$. What can you say about the reduced row echelon form (RREF) of A?
   (i) It has three leading ones
   (ii) It has determinant 3
   (iii) It is I_5
   (iv) It is I_3
   (v) None of the above

   (b) Suppose $\mathbf{v_1}$ and $\mathbf{v_2}$ are solutions to the linear system $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \neq \mathbf{0}$. What linear system is $\mathbf{v_1} + 2\mathbf{v_2}$ a solution to?
   (i) $A\mathbf{x} = 3\mathbf{b}$
   (ii) $3A\mathbf{x} = \mathbf{b}$
   (iii) $A\mathbf{x} = \mathbf{b}$
   (iv) $A\mathbf{x} = \mathbf{0}$
   (v) None of the above

3. Is the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ a linear combination of the vectors

$\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$? Why or why not? (12 pts)

4. Suppose A and B are invertible matrices with $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 1 \end{bmatrix}$ and

$$B^{-1} = \begin{bmatrix} 1 & -4 & 2 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$

(a) Compute $(BA^T)^{-1}$. (12 pts)

(b) Let $\mathbf{c}$ be the vector $\mathbf{c} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$. Find all solutions to the linear system $BA^T\mathbf{x} = \mathbf{c}$. (6 pts)

5. Let $A = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$.

(a) Compute $\det(A)$. (12 pts)

(b) One of the following matrices is A^{-1}, which one is it? Circle your answer. (5 pts)

(i) $\begin{bmatrix} 0 & \frac{1}{5} & 0 & \frac{3}{5} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & \frac{2}{5} & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & \frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 0 & 0 & \frac{3}{5} \\ 0 & 0 & -1 & 0 \\ \frac{2}{5} & 0 & 0 & -\frac{1}{5} \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & \frac{1}{5} & 0 & 0 \\ -2 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -\frac{1}{3} \end{bmatrix}$

(iv) $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{3} \\ \frac{1}{5} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{bmatrix}$

6. Find all solutions to the following linear system using Gaussian elimination or Gauss-Jordan reduction. Make sure to specify the row operations that you are doing and write your answer as a vector. (16 pts)

$$\begin{gathered} x + y - z - 3w = 6 \\ y + 2z = 4 \\ x + y - z - 2w = 7 \\ 2x + y - 4z - 6w = 8 \end{gathered}$$

7. Let A be an $n \times n$ skew symmetric matrix.

(a) Show that if n is odd, then A is not invertible. (12 pts)
Hint: Use determinants and their properties.

(b) Show that part (a) is not true for n even by finding an invertible 2×2 skew symmetric matrix. (5 pts)