MATH 2443
Final Exam Review Sheet

1. Assume that the function $f(u, v)$ has continuous partial derivatives f_u and f_v and suppose that $f_u(1, 1) = 1$ and that $f_v(1, 1) = 2$. A new function $g(x, y, z)$ is defined by setting $g(x, y, z) = f(x/y, y/z)$. Compute $g_y(1, 1, 1)$.

2. Evaluate the limit or show it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4 + y^4}$.

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2(y)}{x^2 + 2y^2}$.

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$.

   (d) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{y - 2x}{4 - xy^2}$.

3. Find all critical points of the function f and determine if each critical point is a local max, local min, or saddle point.

   (a) $f(x, y) = x^3y + 12x^2 - 8y$

   (b) $f(x, y) = e^{4y-x^2-y^2}$

4. Find the point or points on the curve $x^2 + 3y^2 = 36$ which are closest to the point $(2, 0)$. Find the point or points on the curve which are furthest from the point $(2, 0)$.

5. Evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{y^2}{(x^2 + y^2)^{3/2}}\, dydx$.

6. Let $f(x, y, z)$ be differentiable. Suppose $f(1, 3, 5) = 7$ and $\nabla f(1, 3, 5) = \langle 2, -3, 1\rangle$.

   (a) Compute the directional derivative of f at the point $(1, 3, 5)$ in the direction of the point $(-1, 4, 7)$.

   (b) Find the equation of the tangent plane to the surface $f(x, y, z) = 7$ at the point $(1, 3, 5)$.

   (c) Use linear approximation to estimate $f(.9, 3.2, 5.1)$.

   (d) Compute $\nabla g(3, 2)$ where $g(x, y) = f(2y - x, xy - 3, x + y)$.

7. Find the area of the part of the surface $z = x^2 + y^2$ between the planes $z = 1$ and $z = 2$.

8. Evaluate $\int_0^1 \int_1^4 x\sqrt{3+x^2/y}\; dydx + \int_1^2 \int_{x^2}^4 x\sqrt{3+x^2/y}\; dydx.$

9. The function $f(x,y,z) = x^2+2xy+2y^2+3z^2$ has a minimum value on the plane $x+3y+3z=8$. Find the point where the minimum occurs.

10. Let $w = x\sqrt{y} - x - y$. Find the maximum and minimum values of w and where the occur on the triangular region bounded by the x-axis, the y-axis, and the line $x+y=12$.

11. Given a function $f(x,y)$, suppose its gradient at the point $(1,2)$ is $\langle 2,-4\rangle$.

    (a) Find the directional derivative of f in the direction of the origin.

    (b) Find the directional derivative of f in the direction of the maximum rate of increase of f.

    (c) Let $w = f(t^3, t^2+1)$. Find dw/dt at $t=1$.

12. Find $\int_C y^2(e^x+1)dx + 2y(e^x+1)dy$ where C is the closed path formed of three parts: the curve $y=x^2$ from $(0,0)$ to $(2,4)$, the line segment from $(2,4)$ to $(0,2)$ and the line segment from $(0,2)$ to $(0,0)$.

13. A particle is moved in the plane from the origin to the point $(1,1)$. While it is moving, it is acted on by the force $F = \langle y^2 - ye^x + xy, 2xy - e^x + x^2\rangle$. This experiment is done twice. The first time the particle is moved in a straight line and the second time is it moved along the curve $y = x^3$. The work done by the force the first time is W_1 and the second time it is W_2. Determine which of W_1 and W_2 is bigger and by how much.

14. The force $F = \langle e^{x^2}, 2x - e^{y^2}\rangle$ acts on a particle moving from $(0,0)$ to $(1,1)$.

    (a) Compute the work done by the force if the particle moves in a straight line.

    (b) Compute the work done if the particle moves first along the x-axis to $(4,0)$ then then in a straight line to $(1,1)$.

15. Let $w = f(x,y,z)$ be a differentiable function. At the point $x=3, y=2, z=1$ assume that $w=4, \partial w/\partial x = -1, \partial w/\partial y = 2$, and $\partial w/\partial z = 3$. Now view z as a function of x and y implicitly defined by $f(x,y,z)=4$. Find ∇z at $x=3, y=2$.

16. Find the maximum and minimum of $f(x,y,z) = xy + \frac{1}{3}z^3$ on $x^2+y^2+2z^2 \le 32$.

17. Set up but do not evaluate an integral equal to the area of that part of the surface $z = \sqrt{1-x-y}$ that lies inside the cylinder of radius 1 whose axis is the x-axis.

18. Evaluate $\int_C (y+\sin(x))dx + (z^2+\cos(y))dy + x^3 dz$ where C is the curve parametrized by $r(t) = \langle \sin(t), \cos(t), \sin(2t)\rangle$, $0 \le t \le 2\pi$. Hint: C is on the surface $z = 2xy$.

19. (a) Find a number c such that the force field $F = \langle ye^x + 3x^2 + 3y^2, e^x + cxy + 3y^2\rangle$ is conservative.

    (b) Suppose the constant c has the value found in part a. Find a function $f(x, y)$ such that $F = \nabla f$.

    (c) Continuing to assume that c has the value found in part a, find the work done by F on a particle moving from $(1, 0)$ to $(0, 1)$ along the circle of radius 1 centered at the origin.

20. A solid sphere with radius $\sqrt{2}$ is cut into two unequal piece by a plane, where the distance from the center of the ball to the plane is 1 unit. Set up, but do not evaluate, integrals equal to the volume of the smaller piece. Do this in rectangular, spherical, and cylindrical coordinates.

21. Let S be the surface consisting of three surfaces S_1, S_2, S_3 where S_1 is the part of the cylinder $x^2 + y^2 = 16$ with $0 \le z \le 4$, S_2 is the disk $x^2 + y^2 \le 16$ on the plane $z = 4$, and S_3 is the hemisphere $z = \sqrt{16 - x^2 - y^2}$. Find $\iint_S F \cdot\, d\mathbf{S}$ where $F = \langle e^{\cos(z)}, 2y + 3x, 1/(x^2 + y^2)\rangle$.

22. Let S be the surface of the region which is between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ and also above the cone $z = \sqrt{x^2 + y^2}$. Find $\iint_S F \cdot\, d\mathbf{S}$ where $F = \langle 2x^2 z, xyz, y^4\rangle$.

23. Evaluate $\int_C x^2 y dx + \frac{1}{3}x^3 dy + xy dz$ where C is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$ oriented counterclockwise when viewed from above.

24. Let S be the top and 4 sides (but not bottom) of the cube with vertices $(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0), (1, 0, 1), (0, 1, 1), (1, 1, 1)$ oriented outwards. Let $F = \langle xy, z^2 y, x^3 z\rangle$. Compute $\iint_S F \cdot\, d\mathbf{S}$ and $\iint_S \text{curl}(F) \cdot\, d\mathbf{S}$.