MATH 2443
3rd Midterm Review Sheet

1. Evaluate $\int_C xz\ dx - z\ dy + y\ dz$ where C is the line segment from $(1,1,1)$ to $(2,3,4)$.

2. The force field $F(x,y) = \langle e^{x^2}, 2x - e^{y^2} \rangle$ acts on a particle moving from $(0,0)$ to $(1,1)$.

   (a) Compute the work done by the force if the particle moves in a straight line.

   (b) Compute the work done by the force if the particle moves first along the x-axis to $(4,0)$ and then in a straight line to $(1,1)$.

3. Evaluate $\int_C (1+yz)\ dx + (2y+xz)\ dy + (-3x^2)\ dz$ along the path parameterized by $x = t, y = t^2, z = e^t$ for $0 \leq t \leq 1$.

4. Find the mass of that part of the surface $z = xy$ that lies within one unit of the z-axis if the density at the point (x,y) is given by $\delta(x,y) = x^2 + y^2$. Note: The mass of an object is equal to the integral over the object of the density function.

5. A nonuniform piece of wire if bent into the shape of the curve $y = \sin(x)$ between $x = 0$ and $x = \pi$. The density of the wire at the point (x,y) is equal to $1 + y$. Set up, but do not evaluate, an integral equal to the mass of the wire.

6. Find the work done by the force $F(x,y) = \langle 2x\cos(x^2) + e^y, xe^y \rangle$ in moving a particle along a semicircle of radius 1 from $(1,0)$ to $(-1,0)$.

7. A force given by $F(x,y) = \langle y, e^x \rangle$ acts on a particle moving from the point $(0,1)$ to the point $(2,0)$ along the following path: first along the curve $y = e^x$ from $(0,1)$ to $(2,e^2)$ and then along a line from there to $(2,0)$. Find the work done by the force.

8. Evaluate $\int_C 2xe^y\ dx + (3x + x^2e^y)\ dy$ where C is the triangular path from $(0,0)$ to $(1,1)$ to $(2,0)$ and back to $(0,0)$.

9. Let $F(x,y,z) = \langle e^y + ze^x, xe^y - e^z, e^x - ye^z \rangle$.

   (a) Compute the curl and divergence of F.

   (b) Determine if F is conservative. If yes, find f such that $F = \nabla f$, if not explain why.

   (c) Integrate $\int_C F \cdot dr$ along the line segment from $(1,1,1)$ to $(2,2,2)$.

10. Compute $\int \cos(y^2)dx + x(x - 2y\sin(y^2))dy$ along each of the following paths.

(a) The line segment from $(0, 0)$ to $(1, 0)$.

(b) The line segment from $(0, 1)$ to $(0, 0)$.

(c) The line segment from $(0, 1)$ to $(1, 0)$.

11. Let F be the force field $F(x, y) = \langle x, \sqrt{x^2 + y^2} \rangle$. A particle the feels this force starts at the origin. It is moved along the x-axis to the point $(1, 0)$ and then it is moved along a quarter circle centered at the origin until it reaches the point $(0, 1)$. Finally the particle is returned to the origin along the y-axis. Compute the total work done by the force field on the particle during this round trip.

12. A certain force $F = \langle P, Q, R \rangle$ is not completely known. It is known however that $P = yze^{xy} - y^2 + Axz$, $Q = xze^{xy} + 2e^z + Bxy$, and $R = e^{xy} + 3x^2 + Cye^z$ where A, B, C are constants. A particle is moved from $(0, 0, 0)$ to $(1, 1, 1)$ many times along different paths and it is found that the work done by the force is the same each time. Determine values of A, B, C which might explain this result. Compute the work done by the force using these values of A, B, C.

13. A thin hollow shell has the shape of the paraboloid $z = 9 - x^2 - y^2$ for $z \geq 0$. Find the surface area of the shell.

14. A sphere of radius 2 is centered at the origin. Find the area of that part of the sphere that lies above the region on the (x, y)-plane where $x \geq 0, y \geq 0$ and $x^2 + y^2 \leq 1$.

15. Evaluate the line integral $\int_C xy \, ds$ where C is the part of the ellipse $x^2 + 4y^2 = 4$ in the first quadrant.

16. Evaluate the surface integral $\iint_S xy \, dS$ where S is the triangular region with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

17. Let S be the surface with vector equation $r(u, v) = \langle u \cos(v), u \sin(v), v \rangle$, $0 \leq u \leq 2$, $0 \leq v \leq \pi$. See Figure IV on p. 1115 of the textbook for a picture of this surface.

    (a) Find the equation of the plane tangent to S at the point $(0, 1, \pi/2)$.

    (b) Set up, but do not evaluate, and integral for the surface area of S.

    (c) Evaluate the surface integral $\iint_S \sqrt{1 + x^2 + y^2} \, dS$.

    (d) Evaluate the surface integral $\iint_S F \cdot \, d\mathbf{S}$ where $F = \langle y, x, z^2 \rangle$ and S has upward orientation.