MATH 2443

2nd Midterm Review Sheet

1. Let E be the solid below the surface $z = x + 2y^2$ and above the rectangle $R = \{(x, y)|0 \leq x \leq 2, 0 \leq y \leq 4\}$.

   (a) Estimate the volume of E using a Riemann sum with $m = n = 2$ and sample points the midpoint.

   (b) Estimate the volume of E using a Riemann sum with $m = 2, n = 4$ and sample points the lower left corner.

   (c) Find the actual volume of E using an iterated integral.

2. Let D be the region on the xy-plane between the curves $y = |3x|$ and $y = 4 - x^2$. Set up the double integral $\iint_R f(x, y)dA$ as a sum of iterated integrals in 2 different ways (integrating with respect to x first and integrating with respect to y first).

3. Evaluate the following integrals.

   (a)
$$\int_1^{e^2} \int_{\ln(y)}^{2} \frac{e^{x^2}}{y}\, dx\, dy$$

   (b)
$$\iint_R y \sin(x^3)\, dA$$
   where R is the triangle with vertices $(0, 0)$, $(2, 2)$, and $(2, 0)$.

   (c)
$$\int_0^1 \int_0^{\sqrt{x}} xy\, dy\, dx$$

   (d)
$$\iint_R x^{1/3} y\, dA$$
   where R is the quarter disk defined by $x \geq 0, y \geq 0$ and $x^2 + y^2 \leq 1$.

   (e)
$$\int_0^1 \int_y^{\sqrt{2y-y^2}} \frac{1}{\sqrt{x^2 + y^2}}\, dx\, dy$$

   (f)
$$\int_1^2 \int_{x/\sqrt{3}}^{x\sqrt{3}} \frac{1}{(x^2 + y^2)^{3/2}}\, dy\, dx$$

(g)

$$\int_0^8 \int_{x^{2/3}}^4 \frac{x}{\sqrt{x^2+y^3}}\ dy\ dx$$

(h)

$$\iint\limits_R y\ dA$$

where R consists of all points between the positive x-axis and the line $\theta = \pi/3$ that are also inside the circle of radius 2 centered at the origin and outside of the circle of radius 1 centered at the point $(1, 0)$.

4. Fill in the blanks with x, y and z in some order and evaluate.

$$\int_0^1 \int_0^x \int_x^{x+z} z\ d(\ )d(\ )d(\ )$$

5. Let E be the region bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, y = 3x, z = 0$ in the first octant. Set up $\iiint_E z\ dV$ six different ways. Evaluate one of these integrals.

6. Evaluate:

$$\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xz\ dzdxdy$$

7. Find the mass of the solid E with density function $\delta(x, y, z)$ using the formula

$$\text{Mass} = \iiint_E \delta(x, y, z) dV\ .$$

   (a) E is the region between two spheres with common center and radii $\sqrt{2}$ and 1 and with density at (x, y, z) equal to the distance from (x, y, z) to the center of the spheres.

   (b) E is the region bounded by the planes $x = y, x = 2, y = 0, z = 2x$ and $z = 0$ with $\delta(x, y, z) = y$.

8. Let E be the region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the plane $z = 1$. Set up triple integrals for the volume of E in both cylindrical and spherical coordinates. Evaluate both and check that you get the same answer.

9. A spherical ball of radius 2 has a cylindrical hole of radius 1 bored through it. The ball is centered at the origin and the z-axis forms the axis of the hole. Set up integrals in spherical and cylindrical coordinates for the integral of $f(x, y, z) = x^2 + y^2$ over this region. Do not evaluate.