MATH 2443

1st Midterm Review Sheet

1. Let
$$f(x) = \begin{cases} \sqrt{x^2 - y}, & (x,y) \neq (0,-5),(2,-5) \\ 3, & (x,y) = (0,-5),(2,-5) \end{cases}.$$

   (a) Find and sketch the domain of f.

   (b) Find the range of f.

   (c) Where is f continuous?

2. Find the limit or show it does not exist.

   (a)
$$\lim_{(x,y)\to(0,0)} \frac{x^3y + xy^3}{x^2 + y^2}$$

   (b)
$$\lim_{(x,y)\to(0,0)} \frac{x^3y + xy^3}{x^4 + y^4}$$

   (c)
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{1 - e^{x^2+y^2}}$$

3. I have two functions, $f(x,y)$ and $g(x,y)$. I compute f_x and g_x and get two of these three functions: $e^y, 2xy, x+y$. Then I compute f_y and g_y and get two of these three functions: $2x + e^y, xe^y, x^2 + e^y$. Which function in the first list is not one of f_x or g_x and which function in the second list is not f_y or g_y?

4. The equation $x^2yz - z^2 = x^3 - y^2$ determines a surface through the point $(1,2,3)$.

   (a) Find the equation of the tangent plane to the surface at this point.

   (b) Viewing z as an implicitly defined function of x and y near the point $(1,2,3)$, compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $x = 1$, $y = 2$.

5. Given a function $z = f(x,y)$, one of the following three functions is f_x and one is f_y. Identify them. The functions are: $e^x + e^y, e^x + y, xe^y + y$.

6. Let $z = x\cos(y^2) + e^{xy}$. Use differentials to estimate the change in z as x changes from 7 to 7.1 and y changes from 0 to -.1.

7. The two shorter sides of a right triangle are measured then used to calculate the length of the hypotenuse. The error in measurement of the sides is at most 1 %. Use differentials to estimate the maximum percent error in the length of the hypotenuse.

8. Suppose that $w = f(x, y, z)$ is a differentiable function and that $w = 4$ when $x = 1, y = 2, z = 3$. If $f_x(1, 2, 3) = 5, f_y(1, 2, 3) = -1, f_z(1, 2, 3) = 2$, compute a reasonable approximation for $f(.9, 2.1, 3.2)$.

   Suppose now that x, y and z are not really independent and that as x and y vary, z is constrained to move so that $xyz = 6$. As a result, we can view w as a function of x and y and we write $w = h(x, y)$ to denote this function. Compute $h_y(1, 2)$.

9. Find $\frac{\partial w}{\partial s}$ when $s = 1$ and $t = 1$ where

$$w = f(x, y, z) = x^2 + (y\sqrt{5 + \arctan z})\frac{e^{z^3 - \sqrt{y^4+z}}}{\ln(3 + \cos(\sin(z) + y))}$$

   and $x = s^2 + st + t^2$, $y = t^3$, $z = 2st - s^2$.

10. Suppose $w = xy^2 + zx^2$, $x = rs$, $y = s^2$, $z = t^4$, $s = 2t$, $r = e^{t-1}$. Draw a tree diagram and find $\frac{dw}{dt}$ when $t = 1$.

11. Given functions $f(r, s)$ and $g(x, y)$, create a new function by the formula $w = f(y^2, g(x, y))$. Using the following data, compute the values of $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ when $x = 1, y = 2$. (Assume that all partial derivatives are continuous).
    $g(1, 2) = 3,\ g_x(1, 2) = 4,\ g_y(1, 2) = 5,\ f(4, 3) = 6,\ f_r(4, 3) = 7,\ f_s(4, 3) = 2,$
    $f(1, 2) = 1,\ f_r(1, 2) = 3,\ f_s(1, 2) = 9.$

12. Let $f(x, y) = x^2y^2 + axy - y^4$ where a is some constant. The directional derivative of f at the point $(1, 1)$ in the direction of the point $(5, 4)$ is -1.

    (a) Find a.

    (b) What are the maximum and minimum values of the directional derivative of f at the point $(1, 1)$?

    (c) Find a point such that the directional derivative at $(1, 1)$ in the direction of that point is as small as possible.

13. Suppose that the function $f(x, y)$ is differentiable and assume that $f(3, 4) = 7$ and $\nabla f(3, 4) = \langle 3, -2\rangle$.

    (a) Find a reasonable approximation for $f(2.8, 4.1)$.

    (b) Let $h(s, t) = f(s^2 + t, st + 2s)$. Compute $h_s(1, 2)$.

(c) Now suppose that $g(t)$ is a function that has the property that $f(g(t), t^2) = 7$ for all values of t. If $g(2) = 3$, compute $g'(2)$.

14. Let $z = x(e^y + x)$.

    (a) Compute $\partial z/\partial x$, $\partial z/\partial y$, and $\partial^2 z/\partial y^2$.

    (b) Find ∇z at the point $(2, 0)$.

    (c) What is the directional derivative of z at the point $(2, 0)$ in the direction toward $(-1, 4)$.

15. Suppose f is a differentiable function and at the point $(17, -23)$ the directional derivative of f in the direction of the vector $\langle 3, -1\rangle$ is $-\frac{11}{\sqrt{10}}$. At that same point, the directional derivative of f in the direction of $\langle 2, 7\rangle$ is $\frac{31}{\sqrt{53}}$. Find the directional derivative of f at $(17, 23)$ in the direction of $\langle -2, 1\rangle$.

16. Find the point or points on the curve $2y^3 + 9x^2 = 16$ that are closest to the origin.

17. The function $w = x^2 + y - xy$ is defined on the region bounded by the curve $y = 9 - x^2$ and the x-axis. Find the maximum and minimum values of w on this region and the points where they occur.

18. Find the minimum of $w = x^2 + 2y^2 + 3z^2$ on the plane $x + y + z = 1$ and where it occurs.

19. A solid spherical ball of radius 3 is centered at the origin. The temperature at the point (x, y, z) is given by $T(x, y, z) = 4x + 2y + z^2$. Find the maximum and minimum temperatures on the ball and where they occur.

20. Let $f(x, y) = 2x^2y + \frac{1}{2}y^2 - x^4 - 12y$. Find all critical points of f. For each critical point, determine if it is a local max, a local min, or a saddle point.