IMPORTANT THINGS TO KNOW FROM TRIGONOMETRY

1. Trigonometry on the Unit Circle: Consider the unit circle $x^2 + y^2 = 1$. If $(x, y)$ is a point on the unit circle and if $t$ is the angle subtended by the point in the center of the circle then $(x, y) = (\cos t, \sin t)$.

From this it follows that

$$\cos t = x \quad \text{and} \quad \sec t = \frac{1}{x}$$

$$\sin t = x \quad \text{and} \quad \csc t = \frac{1}{y}$$

$$\tan t = \frac{y}{x} \quad \text{and} \quad \cot t = \frac{x}{y}$$

Remark: Note that $\sec t$, $\csc t$ and $\cot t$ are the reciprocals of $\cos t$, $\sin t$ and $\tan t$

2. Converting from Degrees to Radians: To convert the given angle from degrees to radians multiply the given angle by $\frac{\pi}{180}$

3. Converting from Radians to Degrees: To convert the given angle from radians to degrees multiply the given angle by $\frac{180}{\pi}$

4. Arc Length Formula: $S = r\theta$, where $S$ is the length of the arc, $r$ is the radius of the circle and $\theta$ is the angle subtended by the arc in the center of the circle.

Remark: When calculating the arc length make sure the central angle $\theta$ is always in radians. If $\theta$ is given in degrees then convert it to radians before using the arc length formula.
5. Fundamental Identities

\[
\sin^2 t + \cos^2 t = 1
\]
\[
1 + \tan^2 t = \sec^2 t
\]
\[
1 + \cot^2 t = \csc^2 t
\]

6. Period of Trigonometric functions

\[
\sin t, \cos t, \csc t \text{ and } \sec t \text{ are periodic functions of period } 2\pi
\]
\[
\tan t, \cot t \text{ are periodic functions of period } \pi
\]

7. Even and Odd trigonometric functions

\[
\sin(-t) = -\sin(t) \quad [\text{Odd function}]
\]
\[
\cos(-t) = \cos(t) \quad [\text{Even function}]
\]
\[
\tan(-t) = -\tan(t) \quad [\text{Odd function}]
\]

**Remark:** Since \( \csc t \) and \( \cot t \) are the reciprocals of \( \sin t \) and \( \tan t \) they odd functions. \( \sec t \) is the reciprocal of \( \cos t \) and this makes \( \sec t \) an even function.

8. Trigonometric Functions using Right Triangles: We can define trigonometric functions using Right triangles as follows.

\[
\sin t = \frac{\text{Opp}}{\text{Hyp}} \quad \csc t = \frac{\text{Hyp}}{\text{Opp}}
\]
\[
\cos t = \frac{\text{Adj}}{\text{Hyp}} \quad \sec t = \frac{\text{Hyp}}{\text{Adj}}
\]
\[
\tan t = \frac{\text{Opp}}{\text{Adj}} \quad \cot t = \frac{\text{Adj}}{\text{Opp}}
\]
9. Signs of trigonometric Functions

All Trigonometric Functions are positive in the First Quadrant. (i.e $0 < t < \frac{\pi}{2}$)

- $\sin t$, $\csc t$ are positive in the Second Quadrant. (i.e $\frac{\pi}{2} < t < \pi$)
- $\tan t$, $\cot t$ are positive in the Third Quadrant. (i.e $\pi < t < \frac{3\pi}{2}$)
- $\cos t$, $\sec t$ are positive in the Fourth Quadrant. (i.e $\frac{3\pi}{2} < t < 2\pi$)

Remark: You can remember the signs of these functions by remembering the first four letters of this sentence, "A Smart Trig Class".

10. Reference Angles: Reference Angle $\theta'$ is defined to be the acute angle made by the terminal Side of $\theta$ with the $x-axis$.

- $\theta' = \theta$ [if $\theta$ lies in the First Quadrant]
- $\theta' = \pi - \theta$ [if $\theta$ lies in the Second Quadrant]
- $\theta' = \theta - \pi$ [if $\theta$ lies in the Third Quadrant]
- $\theta' = 2\pi - \theta$ [if $\theta$ lies in the Fourth Quadrant]

11. Finding Reference Angles: Follow these steps to find Reference Angles.

Step 1. Check if the given angle $\theta$ lies between 0 and $2\pi$. If yes then find the reference angle by using 10. If no then goto step 2.

Step 2. Check if the given angle $\theta > 2\pi$. If yes then subtract multiples of $2\pi$ till you get an angle which is between 0 and $2\pi$. This tells you the quadrant in which the terminal side of $\theta$ lies. Once you know this you can find the reference angle by using 10. If no then go to step 3.

Step 3. Check if the given angle $\theta < 0$. Then add multiples of $2\pi$ till you get an angle which is between 0 and $2\pi$. This tells you the quadrant in which the terminal side of $\theta$ lies. Once you know this you can find the reference angle by using 10.
12. **Graphing Trigonometric Functions:** The following steps illustrate the method of graphing functions of the following type.

i) \( y = A \sin(Bx - c) + D \)

ii) \( y = A \cos(Bx - c) + D \)

**Step 1.** Compare the given equation with the above functions and determine the constants A, B, C, D.

**Step 2.** Find the Amplitude, Period and the Phase shift using the following.

Amplitude = \(|A|\)

Period = \(\frac{2\pi}{|B|}\)

Phase Shift = \(\frac{C}{B}\)

**Step 3.** Graph the given cosine or sine function in the interval from 0 to \(\frac{2\pi}{|B|}\) ignoring only the shifts.

**Step 4: Horizontal Shifts:** If \(\frac{C}{B} > 0\) take the graph obtained in step 3 and move it to the right by \(\frac{C}{B}\) units. If \(\frac{C}{B} < 0\) take the graph obtained in step 3 and move it to the left by \(\frac{C}{B}\) units.

**Step 5: Vertical Shifts:** If \(D > 0\) take the graph obtained in step 4 and shift it upwards by \(D\) units. If \(D < 0\) take the graph obtained in step 4 and shift it downwards by \(D\) units.

13. **Solution to the problem I missed in class:** Prove the following identity.

\[
\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1} = \frac{\sin x + 1}{\cos x}
\]

\[
\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1} = \left(\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1}\right) \left(\frac{\sin x + \cos x + 1}{\sin x + \cos x + 1}\right)
\]

\[
= \left(\frac{(1 + \sin x)^2 - \cos^2 x}{(\sin x + \cos x)^2 - 1}\right) \text{[Simplifying we get the following]}
\]

\[
= \frac{2\sin^2 x + 2\sin x}{2\sin x \cos x} \quad \text{[Now } 2 \sin x \text{ is a common factor.]}
\]

\[
= \frac{1 + \sin x}{\cos x} \quad \text{[We get this by cancelling the common factors.]}
\]