

① Find y , when $\sin y = -\frac{\sqrt{3}}{2}$ and $0 \leq y < 2\pi$,

(y should be in radians)

$\sin y < 0$ Quad III, IV

$$y = \frac{4\pi}{3}, \frac{5\pi}{3}$$

② Convert your answers to degrees.

$$\left(\frac{4\pi}{3}\right)\left(\frac{180^\circ}{\pi}\right) = \left(\frac{720}{3}\right)^\circ = \boxed{240^\circ}, \quad \left(\frac{5\pi}{3}\right)\left(\frac{180^\circ}{\pi}\right) = \left(\frac{900}{3}\right)^\circ = \boxed{300^\circ}$$

③ What are the reference angles?

$$\text{ref}_{240^\circ} = 240^\circ - 180^\circ \\ = \boxed{60^\circ}$$

$$\text{ref}_{300^\circ} = 360^\circ - 300^\circ \\ = \boxed{60^\circ}$$

④ Find all values of α between 0 and 2π

such that $\cos^2 \alpha = \frac{3}{4}$

$$\sqrt{\cos^2 \alpha} = \sqrt{\frac{3}{4}}$$

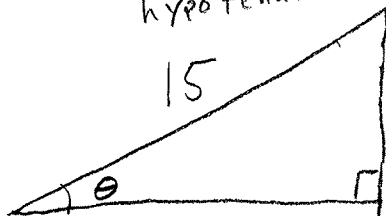
$$\text{So } \cos \alpha = \pm \frac{\sqrt{3}}{2}$$

$$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

⑤ Find the angle θ for the following ^{right} triangle.

hypotenuse

Relate opp and hyp



opposite

$$\sin \theta = \frac{5}{15} \Leftrightarrow \theta = \sin^{-1}\left(\frac{5}{15}\right) \\ = 19.47^\circ \approx \boxed{19^\circ}$$

$$\boxed{\theta = 19^\circ}$$

① $y = \sin(\pi x)$ What is the period?

$y = \sin(bx)$ So $\text{period} = \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$
 $b = \pi$

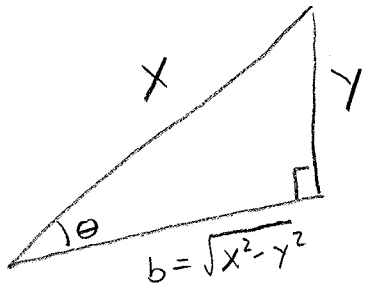
② Find the exact value of $\tan(\sin^{-1}(\frac{y}{x}))$

$\theta = \sin^{-1}(\frac{y}{x}) \Leftrightarrow \sin \theta = \frac{y}{x}$

Pythag. $\Rightarrow x^2 = b^2 + y^2$

$b^2 = x^2 - y^2$

$b = \sqrt{x^2 - y^2}$

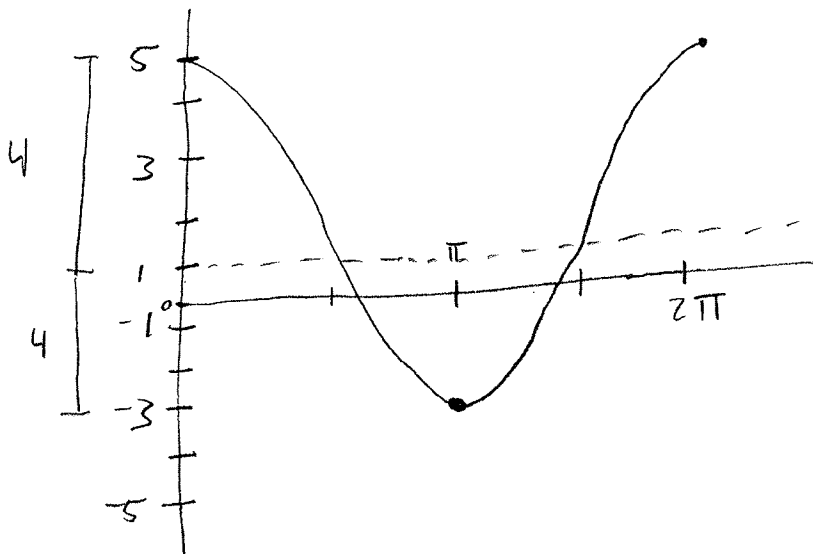


So $\tan(\sin^{-1}(\frac{y}{x})) = \frac{y}{\sqrt{x^2 - y^2}}$

③ What is the amplitude? $y = -7 \cos x - 2$
 7

④ What is the amplitude? $a = \frac{5 - (-3)}{2} = \frac{8}{2} = 4$
What is the vertical shift? up by 1 unit

1



Find the exact values in degrees and radians

① (a) $\sin^{-1}(-1) = -90^\circ$ or $-\frac{\pi}{2}$ radians

(b) $\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$ or $\frac{2\pi}{3}$ rad

(c) $\tan^{-1}(\sqrt{3}) = 60^\circ$ or $\frac{\pi}{3}$ rad

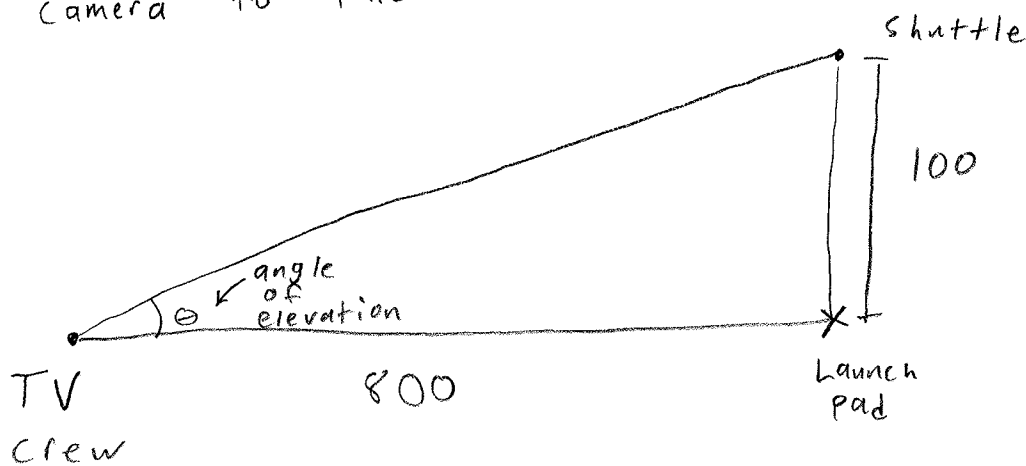
(d) $\cos^{-1}(1.1)$ Undefined! 1.1 is outside the domain.

(e) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$ or $\frac{2\pi}{3}$ rad

(f) $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$ or $-\frac{\pi}{3}$ rad

(g) $\sin^{-1}(1.1)$ undefined! 1.1 is outside the domain

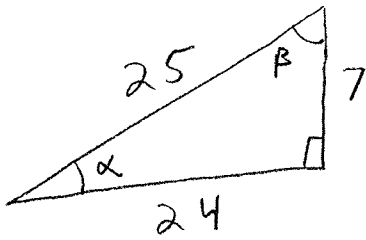
② A television camera at ground level is filming the lift-off of a space shuttle at a point 800 m from the launch pad. When the shuttle is 100 m off the ground, what is the angle of elevation from the camera to the shuttle?



$$\tan \theta = \frac{100}{800} \Leftrightarrow \theta = \tan^{-1}\left(\frac{100}{800}\right) = 7.125^\circ$$

$\theta \approx 7^\circ$

①



Find

- $\sin \alpha = \frac{7}{25}$
- $\csc \alpha = \frac{25}{7}$
- $\cot \alpha = \frac{24}{7}$

- $\cos \beta = \frac{7}{25}$
- $\sec \beta = \frac{25}{7}$
- $\tan \beta = \frac{24}{7}$

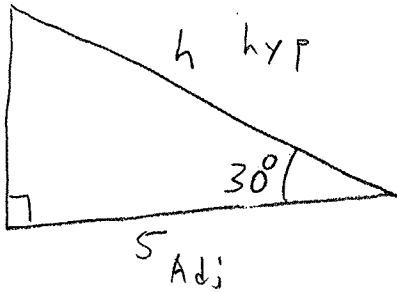
Find the specific angles α and β .

$$\sin \alpha = \frac{7}{25} \Leftrightarrow \alpha = \sin^{-1}\left(\frac{7}{25}\right) = 16^\circ$$

$$\beta = 90^\circ - 16^\circ$$

$$\boxed{\alpha = 16^\circ \quad \beta = 74^\circ}$$

②



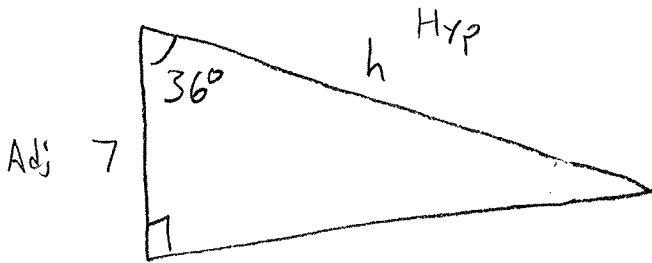
Find h,

Relate Adj and Hyp

$$\cos 30^\circ = \frac{5}{h}$$

$$\boxed{h = \frac{5}{\cos 30^\circ} = 5.77 \approx 6}$$

③

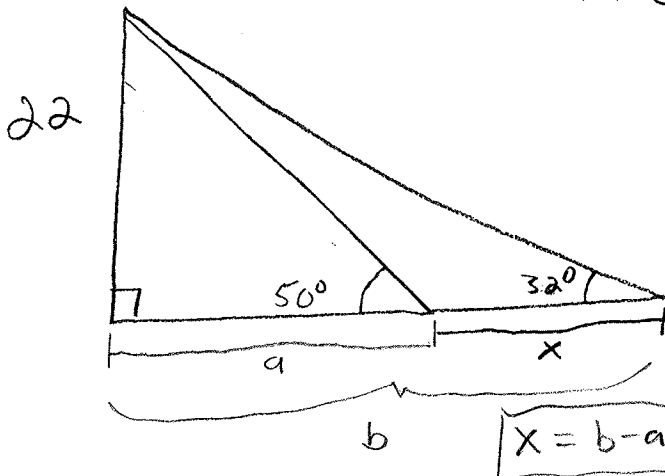


Find h.

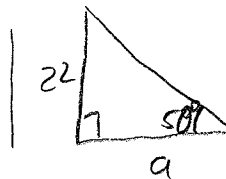
Relate Adj and Hyp

$$\cos 36^\circ = \frac{7}{h} \text{ so } \boxed{h = \frac{7}{\cos 36^\circ} = 8.65 \approx 9}$$

④



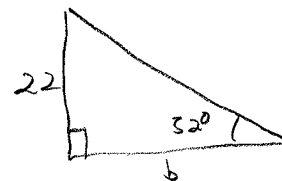
Find x



$$\tan 50^\circ = \frac{22}{a}$$

$$a = \frac{22}{\tan 50^\circ} = 18$$

$$\boxed{a = 18}$$



$$\tan 32^\circ = \frac{22}{b}$$

$$b = \frac{22}{\tan 32^\circ} = 35$$

$$\boxed{b = 35}$$

$$x = b - a = 35 - 18$$

$$\boxed{x = 17}$$

① Find a positive and a negative angle coterminal to 280° .

$$280^\circ + 360^\circ = \boxed{640^\circ}$$

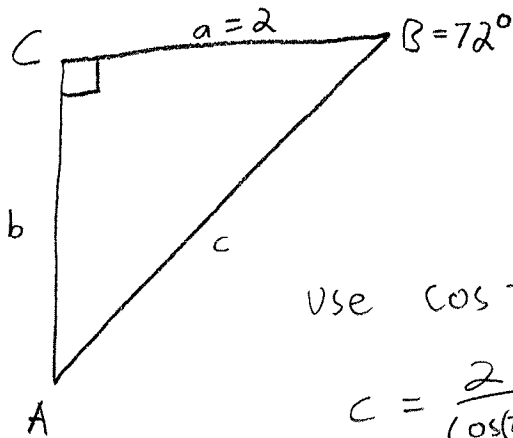
$$280^\circ - 360^\circ = \boxed{-80^\circ}$$

Convert your answers to radians

$$(640^\circ) \left(\frac{\pi \text{ rad}}{180^\circ} \right) = \boxed{\frac{32\pi}{9}}$$

$$(-80^\circ) \left(\frac{\pi \text{ rad}}{180^\circ} \right) = \boxed{-\frac{4\pi}{9}}$$

② Solve the triangle



$$A + B + C = 180^\circ$$

$$A + 72^\circ + 90^\circ = 180^\circ$$

$$A = 90^\circ - 72^\circ = 18^\circ$$

$$\text{Use } \cos 72^\circ = \frac{2}{c} \text{ to find } c$$

$$c = \frac{2}{\cos(72^\circ)} = 6.47 \approx 6$$

$$\text{Pythag. } 6^2 = 2^2 + b^2$$

$$b^2 = 36 - 4$$

$$b = \sqrt{32}$$

$$A = 18^\circ$$

$$B = 72^\circ$$

$$C = 90^\circ$$

$$a = 2$$

$$b = \sqrt{32}$$

$$c = 6$$

③ Find the length of the arc formed by a piece of string wrapped a third of the way around a circle with radius 5 feet.

$\frac{1}{3}$ of the circle is $\frac{2\pi}{3}$ rad. Since a full rotation is 2π and a third of that is $\left(\frac{1}{3}\right)(2\pi) = \frac{2\pi}{3}$

$$S = \theta r = \frac{2\pi}{3} (5 \text{ ft}) = \boxed{\frac{10\pi}{3} \text{ ft}}$$

① Convert $\frac{8\pi}{5}$ into degrees

$$\left(\frac{8\pi}{5}\right)\left(\frac{180^\circ}{\pi}\right) = \boxed{288^\circ}$$

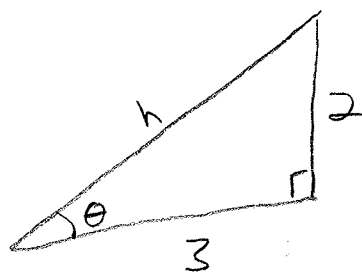
② Find the reference angle for ①.
(You may use the degree answer)

$\theta = 288^\circ$ is in the 4th quadrant so the ref. angle

$$\theta' = 360^\circ - 288^\circ = \boxed{72^\circ}$$

③ Find the exact value of $\sec(\tan^{-1}(\frac{2}{3}))$

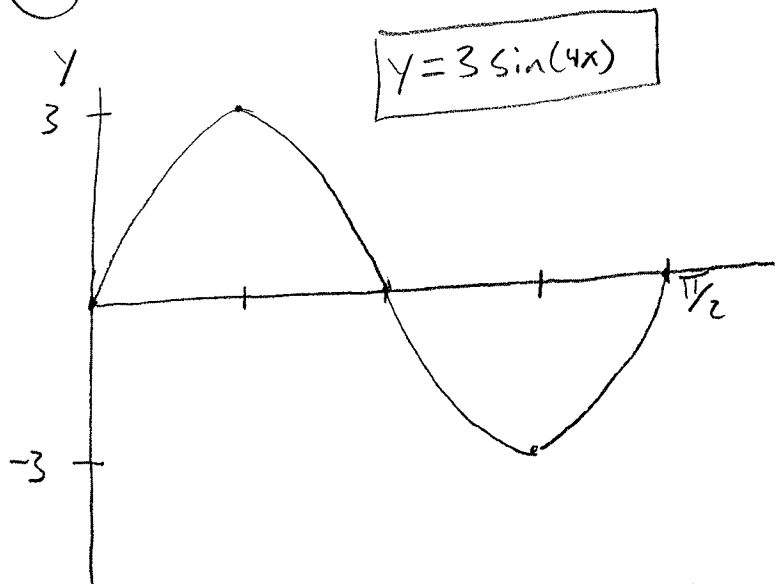
$$\theta = \tan^{-1}(\frac{2}{3}) \Leftrightarrow \tan\theta = \frac{2}{3}$$



$$\begin{aligned} h^2 &= 3^2 + 2^2 \\ h^2 &= 9 + 4 \\ \text{so } h &= \sqrt{13} \end{aligned}$$

$$\text{so } \boxed{\sec(\tan^{-1}(\frac{2}{3})) = \frac{\sqrt{13}}{3}}$$

④ Find the equation of the following graph.



No vert. or horizontal shift.

① Graph does not start at a maximum or minimum, so it can be written as a sine graph

$$\text{② Amplitude} = \frac{3 - (-3)}{2} = \frac{6}{2} = \boxed{3 = a}$$

$$\text{③ period} = \frac{\pi}{2}$$

$$\text{so } \frac{2\pi}{b} = \frac{\pi}{2}$$

$$\begin{aligned} \text{Solve for } b & \quad \frac{4\pi}{\pi} = \frac{b\pi}{\pi} \\ & \quad \boxed{b=4} \end{aligned}$$