1. $48^2 = 24^2 + 31^2 - 2(24)(31)\cos(x)$

Law of Cosines ↗

$767 = -1488\cos(x)$

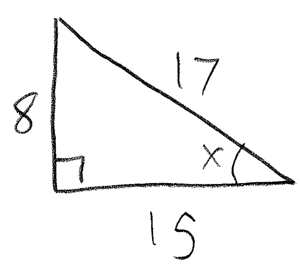
$\cos(x) = \dfrac{-767}{1488}$

$x = \cos^{-1}\left(\dfrac{-767}{1488}\right)$

$\cong 121.028^\circ$

B $\cong 121^\circ$

2. $\sin(2x) + \cos(2x) = (2\sin x\cos x) + \cos^2 x - \sin^2 x$



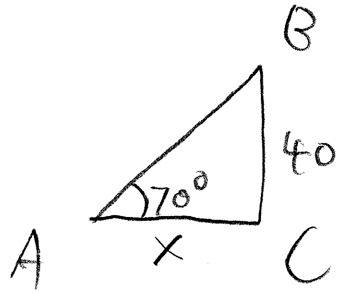
$\sin x = \dfrac{opp}{hyp} = \dfrac{8}{17}$

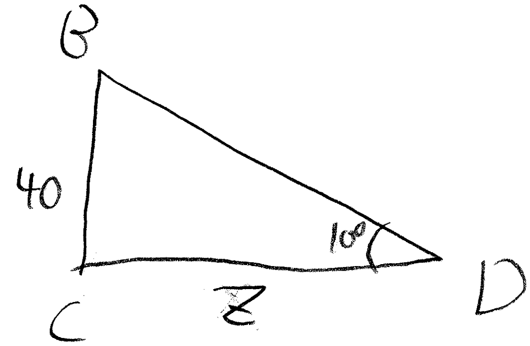
$\cos x = \dfrac{adj}{hyp} = \dfrac{15}{17}$

$= 2\left(\dfrac{8}{17}\right)\dfrac{15}{17} + \left(\dfrac{15}{17}\right)^2 - \left(\dfrac{8}{17}\right)^2$

D $= \dfrac{240}{289} + \dfrac{225}{289} - \dfrac{64}{289} = \dfrac{401}{289}$

3.





$$\frac{40}{x} = \tan 70^\circ$$

$$\frac{40}{\tan 70^\circ} = x$$

$$14.56 \doteq x$$

$$\tan 10^\circ = \frac{40}{z}$$

$$z = \frac{40}{\tan 10^\circ}$$

$$z \doteq 226.85$$

$$AD = 14.56 + 226.85$$

$$= \cancel{241}.41$$

C

4. $\sin x - \cos x = 0$

$\sin x = \cos x$

when does $\sin x = \cos x$? At $x = \frac{\pi}{4}$ & $\frac{5\pi}{4}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$ $\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} = \cos\frac{5\pi}{4}$

$\boxed{\text{Or}}$ $\sin x - \cos x = 0$

$(\sin x - \cos x)(\sin x + \cos x) = 0 \cdot (\sin x + \cos x)$

$\sin^2 x - \cos^2 x = 0$

$\sin^2 x - (1 - \sin^2 x) = 0$

$2\sin^2 x - 1 = 0$

$\sin^2 x = 1/2$

$\sin x = \pm\sqrt{1/2}$

$\sin x = \pm\sqrt{2}/2$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Check solutions to see if $\sin x - \cos x = 0$.
If $x = \frac{3\pi}{4}, \frac{7\pi}{4}$, you get $\pm\sqrt{2} = 0$, so neither of those works.

E

5. $\frac{1-2\sin^2 x}{2\sin x\cos x} = \frac{\cos(2x)}{\sin(2x)} = \cot(2x)$

C

---

6. $(\cos x + \sin x)^2 \stackrel{?}{=} 1$

$$\begin{aligned} \text{L.H.S.} &= \cos^2 x + 2\sin x\cos x + \sin^2 x \\ &= \sin^2 x + \cos^2 x + 2\sin x\cos x \\ &= 1 + 2\sin x\cos x \neq 1 \end{aligned}$$

$\frac{2}{\sec x\csc x} \stackrel{?}{=} \sin 2x$

$$\begin{aligned} \text{L.H.S.} &= 2(\cos x)\sin x \\ &= 2\sin x\cos x \\ &= \sin 2x \end{aligned}$$

B

7. $$\frac{x}{\sin 70^\circ} = \frac{18}{\sin 60^\circ}$$

$$x = \frac{18(\sin 70^\circ)}{\sin 60^\circ}$$

C $$x = 19.53$$

---

8. $$x^2 = 12^2 + 15^2 - 2 \cdot 12 \cdot 15 \cos(140^\circ)$$

$$x^2 = 369 - 360 \cos(140^\circ)$$

$$x^2 \cong 644.776$$

A $$x = 25.39$$

---

9. Heron's area formula

$$s = \frac{17+20+31}{2} = 34$$

$$\text{Area} = \sqrt{34(34-17)(34-20)(34-31)}$$

D $$= \sqrt{24276}$$

$$\cong 155.81$$

10. $\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos(45)\cos(30) + \sin(45)\sin(30)$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

C $\quad = \frac{\sqrt{6}+\sqrt{2}}{4}$

11. $\frac{\cot x}{\csc x} = \frac{\cos x}{\sin x}\cdot \sin x = \cos x$

A

12. $\sin^3 x = \sin x$

$\sin^3 x - \sin x = 0$

$\sin x(\sin^2 x - 1) = 0$

$\sin x = 0$ & $\sin^2 x - 1 = 0$

$x = 0, \pi$

$\sin x = \pm\sqrt{1}$

$\sin x = \pm 1$

D $\boxed{x = 0, \pi/2, \pi, 3\pi/2}$

$x = \pi/2, 3\pi/2$

13. $1 + \dfrac{1-\cos^2 x}{1-\sin^2 x} = ?$

$$= 1 + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

$$= \sec^2 x$$

E

14. $\cos\left(\frac{3\pi}{2} - x\right) = \cos\left(\pi + \left(\frac{\pi}{2} - x\right)\right)$

$$= \cos(\pi)\cos\left(\frac{\pi}{2} - x\right) - \sin(\pi)\sin\left(\frac{\pi}{2} - x\right)$$

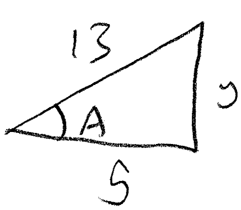
$$= -1 \cdot \cos\left(\frac{\pi}{2} - x\right) - 0 \cdot \sin\left(\frac{\pi}{2} - x\right)$$

$$= -\cos\left(\frac{\pi}{2} - x\right)$$

← cofunction identity, I believe it is called.

A $= -\sin(x)$

15.



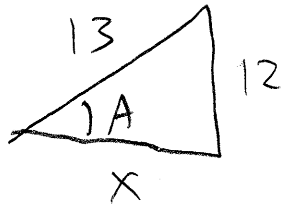
$y^2 + 5^2 = 13^2$

$y = 12$

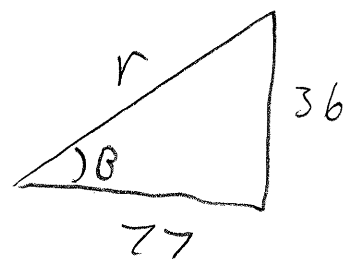
B $\sin(2A) = 2\sin A\cos A = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$

16. A B C D E

1. $\sin(B-A) = \sin(B)\cos(A) - \sin(A)\cos(B)$



$x^2+12^2=13^2$
$x=5$



$r^2=36^2+77^2$
$r^2=7225$
$r=85$

$$= \left(\frac{36}{85}\right)\left(\frac{5}{13}\right) - \left(\frac{12}{13}\right)\left(\frac{77}{85}\right)$$

$$= \frac{-744}{1105}$$

2. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

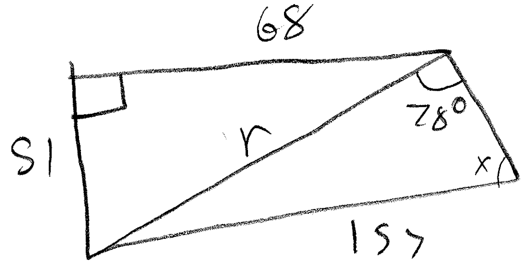
$= \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + (\sin^2 x + \cos^2 x) + 2\sin x\cos x - 2\sin x\cos x$

$= 1 + 1 + 0$

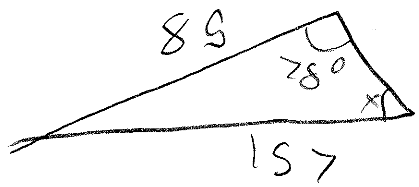
$= 2$

3.



$$51^2 + 68^2 = r^2$$

$$r = 85$$



<u>SSA case</u> (Ambiguous case: maybe 2 solutions)

$$\frac{\sin x}{85} = \frac{\sin(78°)}{157}$$

$$x = \sin^{-1}\left(\frac{85}{157}\sin(78°)\right)$$

$$x = 31.976° \quad \text{or} \quad 180° - 31.976°$$

$$148.02°$$

x can't be 148°, as $148° + 78° > 180°$

$$x = 31.976°$$

$$x \cong 32°$$

4. $4\sin^2 x - 5 = \sin x$

$4\sin^2 x - \sin x - 5 = 0$

$(4\sin x - 5)(\sin x + 1) = 0$

$4\sin x - 5 = 0$ & $\sin x + 1 = 0$

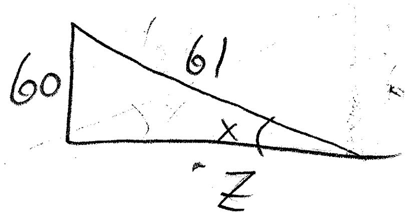
$\sin x = 5/4$ $\quad$ $\sin x = -1$

$\cancel{x \approx}$ $\quad$ ↑ Bigger than 1 $\quad$ $x = 3\pi/2$

No solution from this

$x = 3\pi/2$ Only Solution.

5.



$61^2 = z^2 + 60^2$

$z^2 = 121$

$z = 11$

$$\tan\left(\tfrac{1}{2}x\right) = \frac{\sin x}{1+\cos x} = \frac{60/61}{1 + 11/61} = \frac{5}{6}$$

6. $\frac{1}{2}\left(\cot x + \tan x\right)$

$$= \frac{1}{2}\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$$

$$= \frac{1}{2}\left(\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}\right)$$

$$= \frac{1}{2}\left(\frac{1}{\sin x \cos x}\right)$$

$$= \frac{1}{2\sin x \cos x}$$

$$= \frac{1}{\sin(2x)}$$

$$= \csc(2x)$$