Factoring Polynomials

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1 Special Products

1.1 Difference of two squares: $A^2 - B^2 = (A + B)(A - B)$

1.2 Perfect square trinomials

- $A^2 + 2AB + B^2 = (A + B)^2$
- $A^2 - 2AB + B^2 = (A - B)^2$

1.3 Sum of two cubes: $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

1.4 Difference of two cubes: $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

2 Factoring Trinomials

Strategy for factoring a polynomial $\mathbf{ax^2 + bx + c}$

2.1 Find two first terms whose product is ax^2:

- $(a_1x + \ )(a_2x + \ ) = \underline{\mathbf{ax^2}} + bx + c$ (Where $\mathbf{a_1a_2 = a}$; This is the first step of **FOIL**)

2.2 Find two last terms whose product is c:

- $(a_1x + c_1)(a_2x + c_2) = ax^2 + bx + \underline{\mathbf{c}}$ (Where $\mathbf{c_1c_2 = c}$; This is the **Last** step of **FOIL**)

2.3 Pick outside terms in such a way that the sum of the **Outside** product and the **Inside** product is bx:

- So c_1 and c_2 must satisfy: $a_1x\,(c_2) + a_2x(c_1) = bx$ (This is the sum of the **Outside** and **Inside** products of **FOIL**)
- If no such combination exists then the polynomial is prime

3 Examples

3.1 Factor the following polynomials

3.1.1 $4x^2 - 9 = 2^2x^2 - 3^2$

Now we can use the difference of two squares Let A = 2x and let B = 3
So $4x^2 - 9 = (2x - 3)(2x + 3)$

3.1.2 $x^2 + 6x + 9$

$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$, this is exactly what we need to use the product of perfect square trinomials.
So Let A = x and let B = 3
$x^2 + 6x + 9 = (x + 3)^2$

3.1.3 $b^3 - 8a^6$

$b^3 - 8a^6 = b^3 - 2^3a^6 = b^3 - (2a^2)^3$, this is now what we need to use the sum of two cubes.
Let A = b and Let B = $2a^2$
$b^3 - 8a^6 = (b - 2a^2)(b^2 + 2(b)(2a^2) + [2a^2]^2)$
$= (b - 2a^2)(b^2 + 4a^2b + 4a^4)$

3.1.4 $y^2 + 6y + 8$

This polynomial does not factor using a special product, so we will have to follow the process of factoring trinomials
In this case, a = 1, b = 6 and c = 8.

1. So the two **First** terms are 1 and 1, since 1·1 = 1 (Thus far, we have: (y +)(y +))
2. Find two **Last** terms whose product is 8. There are several possibilities:

c_1	c_2
1	8
-1	-8
2	4
-2	-4

3. The above choices have to satisfy the following equation:

$$\mathbf{y(c_2) + y(c_1) = 6y}$$

- We can rule out negative values since b = 6 is positive.
- We can rule out 1 and 8 by plugging values into our equation:
  $\mathbf{y(8) + y(1) = 9y}$
- The last choice is 2 and 4. Plugging 2 and 4 into our equation provides:
  $\mathbf{y(4) + y(2) = 6y}$

We have found our solution, we just need to plug in the corresponding numbers

$$\mathbf{y^2 + 6y + 8 = (y + 2)(y + 4)}$$

3.1.5 $\mathbf{2x^2y + 3xy - 20y}$

We can see that there is a y in every term. So we can factor out a y:

$2x^2y + 3xy - 20y = y(2x^2 + 3x - 20)$ Now we must factor $2x^2 + 3x - 20$

1. We must find two numbers whose product is 2.
   We only have one choice, 1 and 2
   (Thus far we have: (x +)(2x +))
2. Find two **Last** terms whose product is -20. There are several possibilities:

c_1	c_2
-1	20
20	-1
-20	1
1	-20
2	-10
-10	2
-2	10
10	-2
4	-5
-5	4
-4	5
5	-4

3. Now, we must find c_1 and c_2 such that the following equation is satisfied:
$x(c_2) + 2x(c_1) = 3x$

- By plugging in values, we can rule out all combinations with 10 and 20.
- $x(-5) + 2x(4) = -5x + 8x = 3x$
  So $c_1 = 4$ and $c_2 = -5$ are the values we are looking for. So all other values can be ruled out!

Now $2x^2 + 3x - 20 = (x + 4)(2x - 5)$
Finally, $\mathbf{2x^2y + 3xy - 20y = y(x + 4)(2x - 5)}$

3.1.6 $6a^2 - 5a - 4$

1. We must find two numbers whose product is 6.
   We have two choices, [1 and 6] and [2 and 3].
   First try 2 and 3. If this does not work try 1 and 6.
   (Thus far we have: $(2a + \ )(3a + \ )$)
2. Find two **Last** terms whose product -4. There are several possibilities:

c_1	c_2
1	-4
-4	1
-1	4
4	-1
2	-2
-2	2

3. Now, we must find c_1 and c_2 such that the following equation is satisfied:
$3a(c_1) + 2a(c_2) = -5a$

- $3a(1) + 2a(-4) = 3a - 8a = -5a$
  This is the solution we are looking for. All other values can be ruled out!

Finally, we obtain $\mathbf{6a^2 - 5a - 4 = (2a + 1)(3a - 4)}$

3.1.7 $(x + 2)^{\frac{5}{2}} + (x + 2)^{\frac{3}{2}}$

$(x + 2)^{\frac{5}{2}} + (x + 2)^{\frac{3}{2}} = (x + 2)^{\frac{4}{2}}(x + 2)^{\frac{1}{2}} + (x + 2)^{\frac{2}{2}}(x + 2)^{\frac{1}{2}}$
$= [(x + 2)^2 + (x + 2)](x + 2)^{\frac{1}{2}}$
$=[x^2 + 4x + 4 + x + 2](x + 2)^{\frac{1}{2}}$
$=[x^2 + 5x + 6](x + 2)^{\frac{1}{2}}$ (Use the process of factoring trinomials)
$=(x + 2)(x + 3)(x + 2)^{\frac{1}{2}}$
$=\mathbf{(x + 2)^{\frac{3}{2}}(x + 3)}$