Key

Summer 2014: MATH 2423-271 **Calculus 2** **Bryan Archer**

7/11/2014 Exam 1b 75 minutes

Name:

Instructions.

1. Attempt all questions.
2. Show all steps of your work clearly.
3. Calculators are not allowed.

Question	Points	Your Score
Q1	16	
Q2	10	
Q3	12	
Q4	42	
Q5	10	
Q6	10	
TOTAL	100	

Q1] [16 points]

a) **True/False** (2 point each / 10 points total)

1) $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$ T

2) $\int f(x) \cdot g(x)dx = \int f(x)dx \cdot \int g(x)dx$ F

3) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ F

4) If f is continuous on the interval $[a, b]$, then $\int_a^b f(x)dx$ exists. T

5) If $f(x) \leq g(x)$ on the interval $[a, b]$, then $\int_a^b g(x)dx \leq \int_a^b f(x)dx$. F

b) **Definitions & Theorems** (6 points)

1) **Write the definition of the definite integral of f from a to b** (3 points)

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

2) **Write the definition for an antiderivative of f.** (3 points)

an Antiderivative of f is a function F such that $\frac{d}{dx}F(x) = f(x)$

Q2] [10 points] Let $f(x) = \sin x$ and partition the interval $[0, 6]$ into 3 equally spaced subintervals.

a) (3 points) **What is Δx?**

$$\Delta x = \frac{b-a}{n} = \frac{6}{3} = \underline{2 = \Delta x}$$

b) (3 points) **Write the sample points** $\overline{x_1}, \overline{x_2}$, and $, \overline{x_3}$. ($\overline{x_i}$ is the sample point for the midpoint approximation)

$x_0 = 0$

$x_1 = 2$

$x_2 = 4$

$x_3 = 6$

$\overline{x_1} = 1$

$\overline{x_2} = 3$

$\overline{x_3} = 5$

c) (4 points) **Write out the full expression for M_3, but Do not evaluate!**

$$\boxed{M_3 = \sin(1)\cdot 2 + \sin(3)\cdot 2 + \sin(5)\cdot 2}$$

Q3] [12 points]

a) **Fill in the blanks of the fundamental Theorem of Calculus** (4 points) Let f be a continuous function on the interval $[a, b]$.

1) If $g(x) = \int_a^x f(t)dt$ then g is continuous and differentiable on $[a, b]$ where

$g'(x) =$ $f(x)$

2) If F is any antiderivative of f then

$\int_b^a f(x)dx =$ $F(b) - F(a)$

b) (4 points) **What is g'(x)?**

$$g(x) = \int_3^x \frac{t+11}{t^2+2}dt$$

$$g'(x) = \frac{x+11}{x^2+2}$$

c) (4 points) **What is g'(x)?**

$$g(x) = \int_0^{x^2+1} \cos\theta d\theta$$

$$g'(x) = \cos(x^2+1)\left(\frac{d}{dx} x^2+1\right)$$

$$= \underline{2x\cos(x^2+1)}$$

Q4] [42 points] Compute the definite and indefinite integrals.

a) (7 points) $\int_0^{\pi} 4\cos\theta - 3\sin\theta d\theta$

$$= \cancelto{0}{4\sin\theta\Big|_0^{\pi}} + 3\cos\theta\Big|_0^{\pi}$$

$$= 3(-1-1)\boxed{= -6}$$

b) (7 points) $\int \sec t(\sec t + \tan t)dt$

$$= \int \sec^2 t + \sec t \tan t \, dt$$

$$= \int \sec^2 t \, dt + \int \sec t \tan t \, dt$$

$$\boxed{= \tan t + \sec t + C}$$

Q4] continued

c) (7 points) $\int \frac{x^2 - 3\sqrt{x}}{x^2} dx$

$$= \int \frac{X^2}{X^2} - \frac{3\sqrt{x}}{X^2} \, dx$$

$$= \int 1 \, dx - 3\int X^{-3/2} \, dx$$

$$= X - 3 \frac{X^{-3/2+1}}{-3/2+1} + C$$

$$= X - 3 \frac{X^{-1/2}}{-1/2} + C$$

$$\boxed{\begin{aligned} &= X + 6X^{-1/2} + C \\ &= X + \frac{6}{\sqrt{x}} + C \end{aligned}}$$

d) (7 points) $\int \frac{x}{\sqrt{x^2 - 8}} dx$

$$= \frac{1}{2}\int \frac{1}{\sqrt{u}} \, du = \frac{\cancel{1}}{\cancel{2}} \frac{u^{1/2}}{\cancel{1/2}} + C$$

$$\boxed{= \sqrt{x^2-8} + C}$$

$u = x^2 - 8$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

Q4] continued

e) (7 points) $\int \cos t \cdot \sin(\sin t)dt$ $= \int \sin(\sin t) \cos t \, dt$

$u = \sin t$

$du = \cos t \, dt$

$= \int \sin(u) du$

$= -\cos u + c$

$\boxed{= -\cos(\sin t) + c}$

f) (7 points) $\int_1^2 x\sqrt{x-1}dx$ $= \int_0^1 x \sqrt{u} \, du$

$u = x - 1 \rightarrow u + 1 = x$

$du = dx$

$= \int_0^1 (u+1)\sqrt{u} \, du$

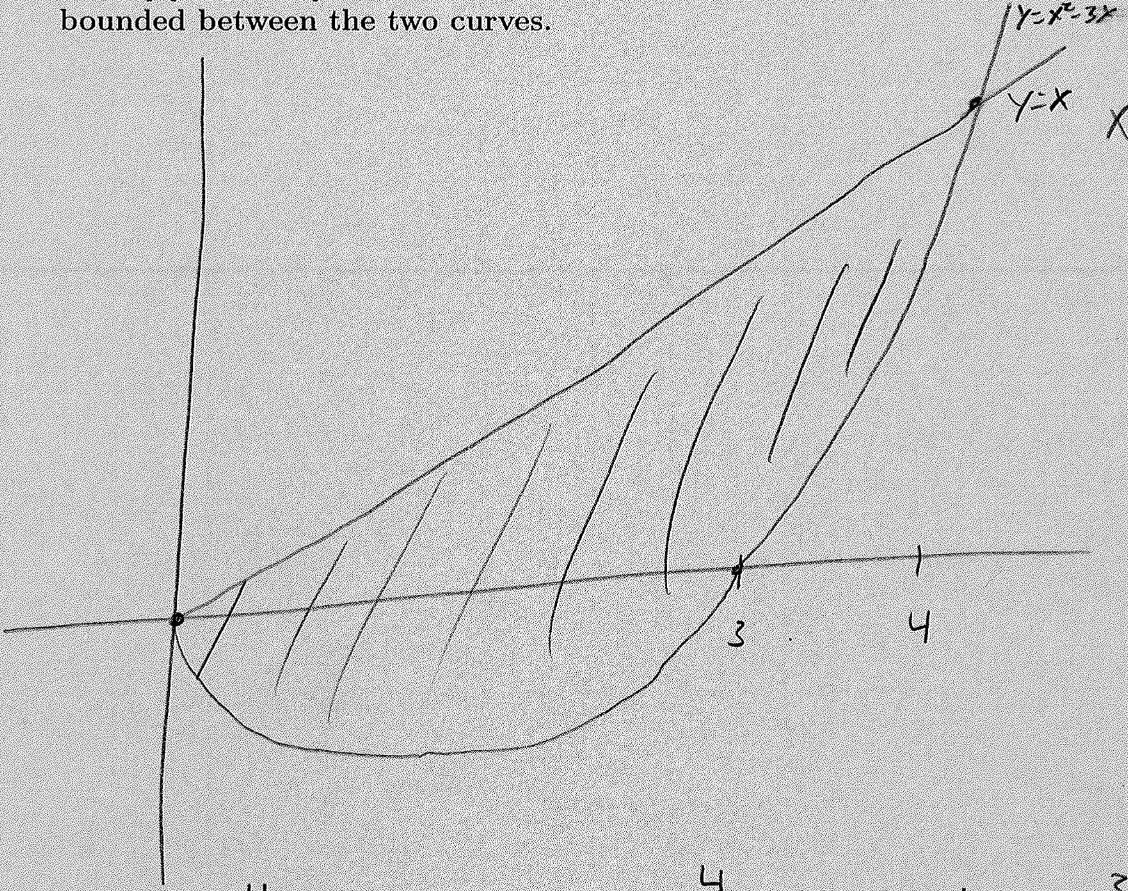
$= \int_0^1 u^{3/2} + u^{1/2} \, du$

$= \frac{2}{5} u^{5/2} \Big|_0^1 + \frac{2}{3} u^{3/2} \Big|_0^1$

$= \frac{2}{5} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{5}{5}$

$\boxed{= \frac{16}{15}}$

Q5] **[10 points] Sketch the Area between the curves $y = x^2 - 3x$ and $y = x$. Find the area bounded between the two curves.**



$x^2-3x=x$

$x^2-4x=0$

$x(x-4)=0$

$x=0,4$

$$A = \int_0^4 x-(x^2-3x)\ dx = \int_0^4 4x - x^2\ dx \qquad \begin{array}{r}32\\ 3\\ \hline 96\end{array}$$

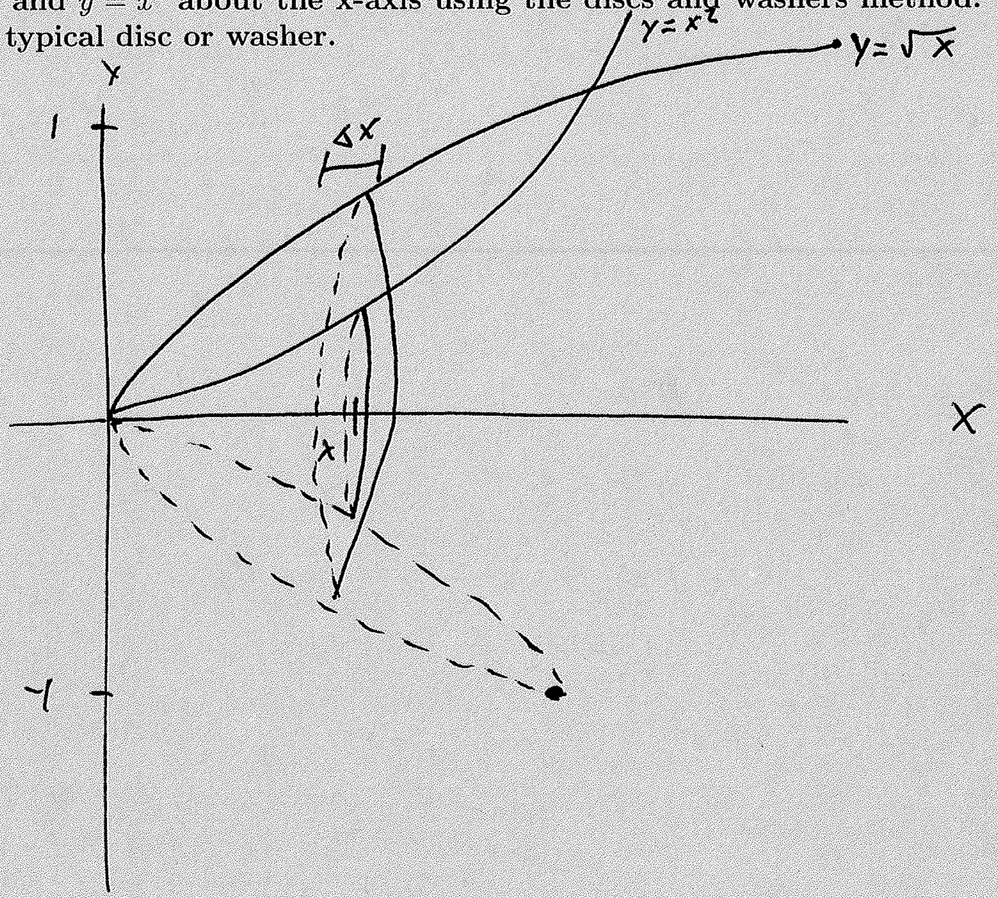
$$= 2x^2\Big|_0^4 - \frac{x^3}{3}\Big|_0^4$$

$$= \frac{3}{3}32 - \frac{64}{3}$$

$$= \frac{96}{3} - \frac{64}{3}$$

$$\boxed{= \frac{32}{3}}$$

Q6] [10 points] Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the x-axis using the discs and washers method. Sketch the region with a typical disc or washer.



⓪ Integrate wrt x

① $r_{out} = \sqrt{x}$, $r_{in} = x^2$, $A(x) = \pi(\sqrt{x})^2 - \pi(x^2)^2$

$= \pi(x - x^4)$

② $V = \int_0^1 A(x)\,dx$

$= \int_0^1 \pi(x - x^4)\,dx$

$= \pi\left[\frac{x^2}{2}\Big|_0^1 - \frac{x^5}{5}\Big|_0^1\right]$

$= \pi\left[\frac{1}{2} - \frac{1}{5}\right]$

$\boxed{= \frac{3\pi}{10}}$