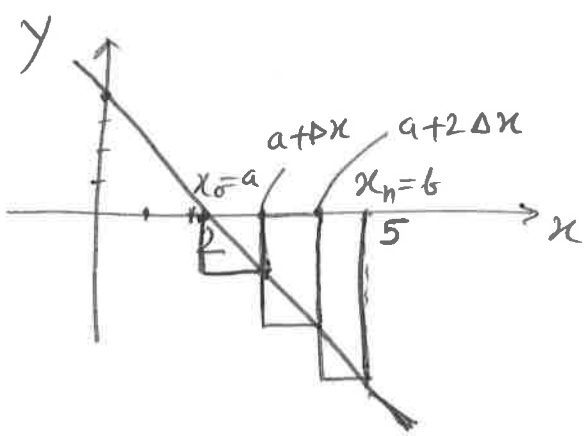
Theorem 4: If f is integrable on $[a,b]$ then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$

<u>4.2/21</u> : $\displaystyle\int_2^5 4-2x\,dx$, $\Delta x = \dfrac{5-2}{n} = \dfrac{3}{n}$,

for right end, $x_i^* = x_i = a + i\Delta x = 2 + i\dfrac{3}{n}$



$$f(x_i^*) = 4 - 2x_i^* = 4 - 2\left(2 + i\frac{3}{n}\right) = -\frac{6}{n}i$$

$$\therefore \int_2^5 (4-2x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\cdot\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} -\frac{6}{n}i\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{-18}{n^2}\cdot\sum_{i=1}^{n} i = \lim_{n\to\infty}\frac{-18}{n^2}\cdot\frac{n(n+1)}{2} = -9.$$

4.2/19 . Express the limit as a definite integral on the given interval.

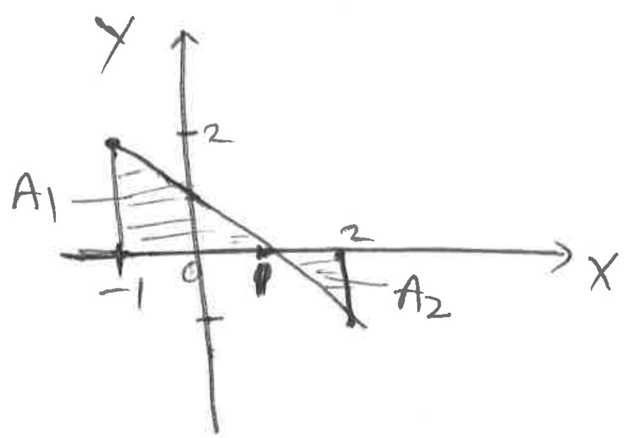
$$\lim_{n\to\infty} \sum_{i=1}^{n} [5(x_i^*)^3 - 4x_i^*]\Delta x \quad , [2,7]$$

$$= \int_2^7 5x^3 - 4x \, dx .$$

- If x_i^* is the left end of the ith interval, then $x_i^* = x_{i-1} = 2 + (i-1)\left(\frac{7-2}{n}\right)$
- If x_i^* is the right end of the ith interval, then $x_i^* = x_i = 2 + i\left(\frac{7-2}{n}\right)$;
- $\Delta x = \frac{7-2}{n}$.

4.2/35 Evaluate the integral by interpreting in terms of area.

$$\int_{-1}^{2} (1-x)\, dx$$

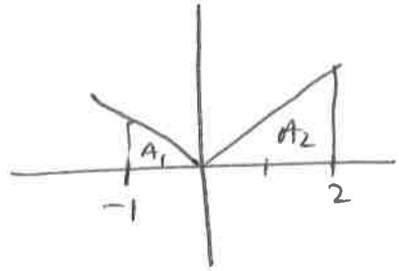


$= A_1 - A_2$

$= \frac{1}{2} \times 2 \times 2 - \frac{1}{2} \cdot 1 \cdot 1$

$= 2 - \frac{1}{2} = \frac{3}{2}$

39. $\int_{-1}^{2} |x|\, dx = A_1 + A_2$



$= \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 2 \times 2$

$= \frac{1}{2} + 2 = \frac{5}{2}$

41. $\int_{\pi}^{\pi} (\text{whatever})\, dx = 0$

43. $\int_0^1 x^2\, dx = \frac{1}{3}$, use this fact and the properties of integral to evaluate

$\int_0^1 (5 - 6x^2)\, dx = \int_0^1 5\, dx - 6 \int_0^1 x^2\, dx = 5 - 6 \times \frac{1}{3}$

$= 5 - 2 = 3$

47. $\int_{-2}^{2} f(x)\, dx + \int_{2}^{5} f(x)\, dx - \int_{-2}^{-1} f(x)\, dx$

$= \int_{-1}^{-2} f(x)\, dx + \int_{-2}^{2} f(x)\, dx + \int_{2}^{5} f(x)\, dx$

$= \int_{-1}^{5} f(x)\, dx.$

4.2. 60. Use property 8 to estimate the value of the integral.

$$\int_0^2 \frac{dx}{1+x^2}$$

Ans: Recall Property 8: If $m \le f(x) \le M$ for $a \le x \le b$

then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

For $x \in [0,2]$, $\frac{1}{1+0^2} \geq \frac{1}{1+x^2} \geq \frac{1}{1+2^2} = \frac{1}{5}$

$$\Rightarrow \quad 1 \geq f(x) \geq \frac{1}{5}$$

$$\Rightarrow \quad \int_0^2 1\,dx \geq \int_0^2 f(x)\,dx \geq \int_0^2 \frac{1}{5}\,dx$$

$$\Rightarrow \quad 1\cdot(2-0) \geq \int_0^2 \frac{dx}{1+x^2} \geq \frac{1}{5}\cdot(2-0)$$

$$\Rightarrow \quad 2 \geq \int_0^2 \frac{dx}{1+x^2} \geq \frac{2}{5}.$$