MATH 3333

Midterm II

November 14, 2008

Name :

I.D. no. :

- Calculators are not allowed. The problems are set so that you should not need calculators at all.

- Show as much work as possible. Answers without explanation will not receive any credit.

- If you preform any row or column operations in a problem, record them using standard notations.

- Best of Luck.

i) Let $L : R_3 \to R_4$ be defined by

$$L(\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}) = \begin{bmatrix} u_1 - u_3 & u_2 + 2u_1 & -u_3 & u_1 + u_3 \end{bmatrix}$$

a) (10 Points) Show that L is a linear transformation.

b) (10 Points) Find a basis for and dimension of the Kernel of L.

c) (10 Points) Find a basis for and dimension of the Range of L.

d) (15 Points) Let $S = \{\begin{bmatrix}1 & 0 & 1\end{bmatrix}, \begin{bmatrix}2 & 1 & 0\end{bmatrix}, \begin{bmatrix}0 & -1 & 1\end{bmatrix}\}$ be a basis for R_3 and T be the natural basis for R_4. Find the matrix representing L with respect to the ordered basis S and T.

ii) Let $V = M_{22}$. Let W be the subset of V consisting of matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a + d = 0$.

a) (10 Points) Show that W is a subspace of V.

b) (15 Points) Find a basis for and dimension of W.

iii) (15 Points) Let V be the set of all odd integers. Define the operations $\oplus$ and $\odot$ as follows:

$$\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}(\text{ usual multiplication}) \text{ and } c \odot \mathbf{u} = \mathbf{u}$$

Is V a vector space ? If Yes, then show that **all** the properties are satisfied. If No, then list **all** the properties that are not satisfied.

iv) (15 Points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Is Rank$(A) = 3$? Explain your answer. Also, find Nullity(A).