Math 3333, Quiz III, October 2, 2007

Name :

1. (7 Points) If A is a non-singular matrix such that $A^2 = A$, then what is $\det(A)$?

   Solutions 1

$$A^2 = A \Rightarrow \det(A^2) = \det(A) \Rightarrow \det(A).\det(A) = \det(A)$$
$$\Rightarrow \det(A)(\det(A) - 1) = 0 \Rightarrow \det(A) = 0 \text{ or } 1.$$

   But A is non-singular and hence $\det(A) \neq 0$. So $\det(A) = 1$.

   Solution 2 Since A is non-singular we know that A^{-1} exists. So,

$$A^2 = A \Rightarrow A^{-1}A^2 = A^{-1}A \Rightarrow A = I_n \Rightarrow \det(A) = 1.$$

2. (7 Points) Find all values of t such that $\det(\begin{pmatrix} t-4 & 0 & 2 \\ 3 & t & -5 \\ 0 & 0 & t+3 \end{pmatrix}) = 0.$

   The determinant of the above matrix is $(t-4)t(t+3)$. Hence the values of t for which the determinant is zero are $t = 4, 0, -3$.

3. (6 Points) State which of the following permutations of $S = \{1, 2, 3, 4, 5, 6, 7\}$ are even and which are odd. Explain.

$$(a)\ 2453671 \quad (b)\ 5237614 \quad (c)\ 6214357$$

   2453671 is an **even** permutation since there are 8 inversions. 5237614 is an **odd** permutation since there are 11 inversions. 6214357 is an **odd** permutation since there are 7 inversions.