Math 3333, Quiz II, September 11, 2007

Name :

1. (8 Points) Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 5 & 4 & -6 \\ -4 & 8 & 0 \end{bmatrix}$. Find a **symmetric** matrix B and a **skew-symmetric** matrix C such that $A = B + C$.

   For any matrix A we know that $\frac{1}{2}(A + A^T)$ is symmetric and $\frac{1}{2}(A - A^T)$ is skew-symmetric and $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$. Hence

$$B = \frac{1}{2}(A + A^T) = \frac{1}{2}\left(\begin{bmatrix} 2 & 1 & 0 \\ 5 & 4 & -6 \\ -4 & 8 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 5 & -4 \\ 1 & 4 & 8 \\ 0 & -6 & 0 \end{bmatrix} \right) = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 4 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$C = \frac{1}{2}(A - A^T) = \frac{1}{2}\left(\begin{bmatrix} 2 & 1 & 0 \\ 5 & 4 & -6 \\ -4 & 8 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 5 & -4 \\ 1 & 4 & 8 \\ 0 & -6 & 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & -2 & 2 \\ 2 & 0 & -7 \\ -2 & 7 & 0 \end{bmatrix}$$

2. (6 Points) Consider the matrix transformation $f(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$. Determine whether $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ and $\mathbf{w}_2 = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}$ are in the range of f ? Explain your answer.

   Let $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$. Then $f(\mathbf{u}) = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ 2x + 3y \\ x + y \end{bmatrix}$. If $f(\mathbf{u}) = \mathbf{w}_1$ then we have $\begin{bmatrix} y \\ 2x + 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$. We can solve this system of linear equations to get a solution $x = 1, y = 1$. Hence $\mathbf{w}_1$ is in the range of f. If $f(\mathbf{u}) = \mathbf{w}_2$ then we have $\begin{bmatrix} y \\ 2x + 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}$. The first two equations gives us $y = 2$ and $x = 1/2$, but these values of x and y do not satisfy the third equation $x + y = 1$. Hence we cannot find a solution to the system of linear equations. So, $\mathbf{w}_2$ is not in the range of f.

3. (6 Points) Determine which of the following matrices are in row echelon form or reduced row echelon form or neither.

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 5 & 6 \\ 0 & 0 & 1 & -3 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 & -7 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

   A is in reduced row echelon form, B is neither and C is in row echelon form.