

**Homework 4** : This homework is due on **September 27**.

1. Find the number of inversions in each of the following permutations of  $S = \{1, 2, 3, 4, 5, 6\}$ . Also state whether the permutations are even or odd.

(a) 134256 (b) 253614 (c) 432516 (d) 632514 (e) 352461

2. Evaluate the determinant of the following matrices

$$A = \begin{pmatrix} 4 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 0 & 5 \end{pmatrix} \quad B = \begin{pmatrix} t-2 & 3 \\ 2 & t-1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 3 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & -1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

3. Find all the values of  $t$  for which the determinant of the matrix  $A = \begin{pmatrix} t-1 & -1 & -2 \\ 0 & t & 2 \\ 0 & 0 & t-3 \end{pmatrix}$  is zero.

4. If determinant of  $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$  is 5 then what is the determinant of the matrix  $B = \begin{pmatrix} a_1 - \frac{1}{2}a_3 & 2a_2 & a_3 \\ b_1 - \frac{1}{2}b_3 & 2b_2 & b_3 \\ c_1 - \frac{1}{2}c_3 & 2c_2 & c_3 \end{pmatrix}$ .

5. If determinant of  $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$  is 3 then what is the determinant of the matrix  $B = \begin{pmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{pmatrix}$

6. Compute the determinant of the following matrices via reduction to triangular form

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -3 & 5 \\ 5 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 2 & 3 & -4 \\ 3 & -2 & 1 & 5 \\ -2 & 0 & 1 & -3 \\ 8 & -2 & 6 & 4 \end{pmatrix} \quad D = \begin{pmatrix} 4 & 1 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 2 \end{pmatrix}$$