Math 3113, Final
May 10, 2007

Name:

ID No:

- Calculators are not allowed. The problems are set so that you should not need calculators at all.

- Show as much work as possible. Answers without explanation will not receive any credit.

- Best of Luck.

1. Consider the initial value problem

$$(x^2 - 3)y'' + 2xy' = 0; \qquad y(0) = 1, y'(0) = 2.$$

(a) (5 Points) Is $x = 0$ an **ordinary** point or **regular singular** point or **irregular singular** point? Explain your answer.

(b) (15 Points) Substitute $y(x) = \sum_{n=0}^{\infty} c_n x^n$ in the differential equation and obtain a recurrence relation for the coefficients c_n.

(c) (15 Points) Solve the recurrence relation derived in part (b) and obtain a general formula for c_n.

(d) (5 Points) Write the solution of the IVP using the given initial values and find the **guaranteed** radius of convergence.

2. (10 Points) Identify the following series in terms of familiar functions.

(a) $f(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!} x^n$

(b) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{(2n+1)!} x^{2n+1}$

3. (10 Points) Identify whether $x = 0$ is an **ordinary** point or **regular singular** point or **irregular singular** point of the following differential equations. Explain your answer.

(a) $x^3 y'' + 2x \sin(x) y' + 31xy = 0$

(b) $2x^4 y'' + 3x^3 y' + (2 + x^2) y = 0$

4. Consider the differential equation

$$x^2y'' + \frac{1}{2}(x + x\cos(x))y' - \frac{1}{4}(1 + x^2)y = 0$$

(a) (15 Points) Find the **exponents** of the Frobenius series solution of the above differential equation.

(b) (5 Points) Will you get one Frobenius series solution or two linearly independent Frobenius series solutions ? Explain your answer.

5. (20 Points) Use the **convolution product theorem** to obtain

$$\mathcal{L}^{-1}\{\frac{1}{(s^2+9)s}\}$$

Do **not** use Partial Fractions method.

6. (20 Points) Suppose $f(t)$ is a periodic function with period $p = 4$ and

$$f(t) = \begin{cases} 4, & \text{if } 0 \leq t < 2; \\ 2, & \text{if } 2 \leq t < 4 \end{cases}$$

Find $\mathcal{L}\{f(t)\}$.

7. Suppose $f(t) = te^t$.

(a) (5 Points) Find $\mathcal{L}\{f(t)\}$.

(b) (7 Points) Show that $f^{(n)}(0) = n$ for all n.

(c) (8 Points) Find a general formula for $\mathcal{L}\{f^{(n)}(t)\}$.

8. Consider the boundary value problem

$$y'' + \lambda y = 0; \qquad y'(0) = 0, y'(\pi) = 0.$$

It is known that there are no eigenvalues for $\lambda < 0$.

(a) (10 Points) Determine whether $\lambda = 0$ is an eigenvalue.

(b) (10 Points) Find all the positive eigenvalues and the associated eigenfunctions.

9. (20 Points) Suppose \$1000 were deposited in a bank at 6% interest compounded continuously. How much will the account contain in 20 years. (Use $e^{1.2} = 3.32011$)

10. (20 Points) Consider $xy'' - y' = 0$. This is a second order differential equation with the variable y missing. **Reduce** to a first order differential equation by making a suitable substitution and then obtain the general solution.