

MATH 3113

Midterm II

November 14, 2008

Name :

I.D. no. :

- Calculators are not allowed. The problems are set so that you should not need calculators at all.
- Show as much work as possible. Answers without explanation will not receive any credit.
- Best of Luck.

i) a) (8 Points) Find

$$\mathcal{L}^{-1}\left\{\frac{4s + 7}{s^2 + 2s + 10}\right\}$$

b) (12 Points) Find the Laplace transform of the function $f(t)$ given by the following graph.

ii) a) (15 Points) Find the partial fractions decomposition of

$$\frac{5s^2 + 2s + 1}{(s + 1)(s^2 + 1)}$$

b) (5 Points) Find

$$\mathcal{L}^{-1}\left\{\frac{5s^2 + 2s + 1}{(s + 1)(s^2 + 1)}\right\}$$

iii) (20 Points) Using the **Laplace transform method**, find the solution to the following initial value problem

$$x'' + x = 4e^{-x}, \quad x(0) = 5, x'(0) = -3$$

Hint: Solution to problem (ii) may come in handy.

iv) (15 Points) Find the Laplace transform of

$$f(t) = \begin{cases} \sin(t), & \text{if } 0 \leq t < 2\pi; \\ 0, & \text{if } 2\pi \leq t < 4\pi; \\ \cos(t), & \text{if } 4\pi \leq t < 6\pi; \\ 0, & \text{if } 6\pi \leq t. \end{cases}$$

v) (10 Points) Find the singular points and guaranteed radius of convergence of a power series solution in powers of $x - 1$ of the differential equation

$$(3 + x^2)y'' + (2x + 1)y' + 4xy = 0.$$

vi) (20 Points) Find the recurrence relation and the first 3 non-zero terms of each of the 2 linearly independent power series solutions of the differential equation

$$(2 - x^3)y'' + x^4y = 0.$$