MATH 3113

Final

December 19, 2008

Name :

I.D. no. :

- Calculators are not allowed. The problems are set so that you should not need calculators at all.
- Show as much work as possible. Answers without explanation will not receive any credit.
- Best of Luck.

i) Solve the initial value problem

$$y'' - 2y' + 2y = 1 + x, \qquad y(0) = 3, y'(0) = 0$$

using the following steps.

a) (10 Points) Find the solution of the homogeneous differential equation

$$y'' - 2y' + 2y = 0.$$

b) (10 Points) Using Part (a), find a particular solution of

$$y'' - 2y' + 2y = 1 + x$$

using either method of undetermined coefficients or variation of parameters.

c) (10 Points) Using Parts (a) and (b), and the initial conditions, find the solution to the IVP.

ii) Consider the boundary value problem

$$y'' + \lambda y = 0, \qquad y(0) = 0, y(\pi) = 0.$$

a) (10 Points) Determine whether $\lambda = 0$ is an eigenvalue. If yes, find the associated eigenfunction.

b) (20 Points) Find all the **positive** eigenvalues and the associated eigenfunctions.

iii) (20 Points) Consider the differential equation

$$xy'' + y' = 4x.$$

This is a second order differential equation with the variable y missing. Reduce to a first order differential equation by a suitable substitution and then obtain the general solution. Alternatively, you can use any other applicable method to solve the differential equation.

iv) (20 Points) State whether $x = 0$ is an ordinary point or regular singular point or irregular singular point for the following differential equations. Explain your answer.

a) $x(1+x)y'' + 2y' + 3xy = 0$.

b) $(x^3 - x)y'' + x\sin(x)y' + x^2y = 0$.

c) $x^2y'' + (x^2 + x)y' + \cos(x)y = 0$.

d) $x^4y'' + x^2y' + y = 0$

v) Consider the differential equation

$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0.$$

The point $x = 0$ is a regular singular point.

a) (10 Points) Find the indicial equation and the indices r_1 and r_2.

b) (20 Points) If you did Part (a) correctly, then you should have obtained $r_1 - r_2$ is a positive integer. Take

$$y(x) = x^{r_2} \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+r_2}$$

and obtain two linearly independent Frobenius series solutions.

.

vi) a) (15 Points) Let a, b be two real numbers. Find the partial fractions decomposition of

$$\frac{as + (b + 8a)}{(s + 3)(s + 5)}$$

b) (15 Points) Using Laplace Transforms, find the solution of the initial value problem

$$x'' + 8x' + 15x = 0, \qquad x(0) = a, x'(0) = b.$$

(**Hint.** You may need to use the result from Part (a))

vii) (20 Points) Using the formula

$$\mathcal{L}\{\frac{f(t)}{t}\} = \int_s^{\infty} F(\sigma) d\sigma$$

find the Inverse Laplace transform of

$$F(s) = \frac{2s}{(s^2+1)^2}.$$

viii) (20 Points) Fill in the following table of Laplace transforms.

$f(t)$	$F(s)$
	$\frac{k}{s^2-k^2}$
$e^{at}f(t)$	
$u(t-4)$	
	$\frac{F(s)}{s}$
$-tf(t)$	