Instructions Be sure to give thorough explanations of your work to guarantee getting full credit.

1. (15 points) Consider the function $g(x, y) = (y + 4x^2)^2$.
   (a) Are there any values of $k$ for which the level curve for $g(x, y)$ at $k$ is the empty set? If so describe them.
   (b) Sketch a contour map for $g(x, y)$. Include at least five different $k$ values.
   (c) Is there a level curve for $g(x, y)$ which passes through the point $(3, -5)$? If so, what is the $k$ value for that level curve?
2. (20 points) Let \( f(x, y) = (x - 2xy + y^2 - 1)^2 \) and let \( P = (-1, 2) \).
   (a) Compute the gradient of \( f(x, y) \).
   (b) Find a normal vector for the plane tangent to the graph of \( f(x, y) \) at \( P \).
   (c) Determine the unit vector \( \mathbf{u} \) for which the directional derivative at \( P \) is maximal, and the unit vector \( \mathbf{v} \) for which the directional derivative at \( P \) is minimal.
   (d) Find the directional derivative for \( f \) at \( P \) in the direction of the vector \( \frac{3}{5} \mathbf{i} - \frac{4}{5} \mathbf{j} \).

3. (15 points) Consider the function \( g(x, y) = e^{y/x} \).
   (a) Describe the domain of \( g \).
   (b) Find an equation for the plane tangent to the graph of \( g(x, y) \) at the point \((1, 2, e^2)\).
   (c) Describe and sketch the graph in the \( xy \)-plane of the level curve for \( g(x, y) \) at \( k = e^2 \).
4. (15 points) (a) Find an equation of the tangent plane to the surface with equation $x^3yz^2 + z = 4$ at the point $(-1, -1, 2)$.
(b) Are there any points on the surface $x^3yz^2 + z = 4$ which have horizontal tangent planes? Explain.

5. (15 points) Use the chain rule to find the partial derivatives $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$ where

$$g(x, y) = xy^3 - 4x^2, \quad x = e^{u^2}, \quad y = \sqrt{v^2 + 1} \sin(u).$$
6. (20 points) Let \( f(x, y) = 3x + 12y - x^3 - y^3 \). Find all critical points for \( f \) and classify each one as local maximum, local minimum or saddle point.

7. (BONUS) (5 points) Does the function \( f(x, y) \) in problem 6 have an absolute maximum or an absolute minimum? Explain.