1. Sketch some level curves for the scalar function \( f(x, y) = x^2 + 9y^2 \) and in the same picture sketch the two dimensional vector field \( \nabla f(x, y) \).

2. Sketch a picture of the two dimensional vector field \( \frac{-y}{\sqrt{x^2 + y^2}} \mathbf{i} + \frac{x}{\sqrt{x^2 + y^2}} \mathbf{j} \) and compare it to the picture of the two dimensional vector field \( \frac{x}{\sqrt{x^2 + y^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}} \mathbf{j} \).

3. For each of the following, find a potential function \( f(x, y) \) for the given vector field or determine that no potential function exists.
   (a) \( \mathbf{F}(x, y) = \frac{(x + y)}{1 + x^2 + y^2} \mathbf{i} + \frac{(x - y)}{1 + x^2 + y^2} \mathbf{j} \)
   (b) \( \mathbf{F}(x, y) = (4x + y, x + 2y - 3) \)
   (c) \( \mathbf{F}(x, y) = (1 + e^{xy} + y + xe^{xy}) \mathbf{i} + (x + x^2e^{xy} + 2y) \mathbf{j} \)

4. Find a scalar function \( f(x, y, z) \) such that \( \mathbf{F}(x, y, z) = \nabla f(x, y, z) \) where \( \mathbf{F}(x, y, z) = (y^2 \cos(z), 2xy \cos(z) - \cos(y), -xy^2 \sin(z)) \).

5. Compute each of the line integrals \( \int_C \mathbf{F} \cdot d\mathbf{r} \):
   (a) where \( \mathbf{F}(x, y) = x^2 \mathbf{i} - 2xy \mathbf{j} \) and \( C \) is the line segment from \((1, 0)\) to \((3, 2)\).
   (b) where \( \mathbf{F}(x, y) = \frac{-y}{\sqrt{x^2 + y^2}} \mathbf{i} + \frac{x}{\sqrt{x^2 + y^2}} \mathbf{j} \) and \( C \) is the circle of radius 5 centered at the origin traversed once around counterclockwise starting at \((5, 0)\).

6. Find a potential function for the vector field \( \mathbf{F}(x, y) = (2xy - y^2, x^2 - 2xy) \) and use it to compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C : x = t^3 - 2, y = t^2 + t - 2, -1 \leq t \leq 1 \).