An $\mathbb{R}$–tree is a metric space $(X,d)$ in which every pair of points is connected by a unique embedded path, and every embedded path is isometric to a closed Euclidean interval. This definition was first used by Tits in the 1970’s to study $SL_2(K)$ for a valued field $K$. In particular he deduced algebraic properties of these groups by studying how they act on certain $\mathbb{R}$–trees. At roughly the same time, $\mathbb{R}$–trees were used to construct examples in the study of length functions on groups initiated by Lyndon. Since then the concept has arisen in many other contexts. Work of Morgan and Shalen showed that $\mathbb{R}$–trees occur naturally in describing compactifications of isometric actions of a fixed group on a hyperbolic space. They are also examples of spaces which are negatively-curved in the sense which Cannon, Gromov and others have brought into prominence. In fact $\mathbb{R}$–trees are essentially the 0-hyperbolic metric spaces, and as such they play a fundamental role and provide important examples in the Gromov theory of negatively curved groups. They also arise in describing the dynamics of the outer automorphisms of free groups. A high point in the development of the general theory of $\mathbb{R}$–trees is Rips Theorem which classifies the groups which act freely on $\mathbb{R}$–trees. The proof of this result is based on examining measured laminations on certain 2-dimensional spaces. In addition, combinatorial trees (which are connected graphs that contain no circuits) are examples of $\mathbb{R}$–trees, and the study of group actions on these spaces has played a central role in combinatorial and geometric group theory since the 1960’s via what is commonly referred to as Bass-Serre theory.

So the theory of $\mathbb{R}$–trees is a topic which lies at the interface of areas such as: geometric topology, low-dimensional topology, geometry of manifolds, hyperbolic geometry, and group theory. This course should be useful for students who plan to specialize in any of these areas of study, and it should also provide some good introductory vistas for students who might like to learn about these areas of study. The prerequisites for the course consist of the first-year graduate topology sequence and the first semester of the graduate algebra sequence.

In the course we will develop the basic theory of $\mathbb{R}$–trees starting from elementary principles, and examine a variety of applications. We will use Chiswell’s book *Introduction to $\Lambda$–Trees* as a main text but also refer to other sources for applications, such as the surveys by Bestvina and Shalen listed below.

References: