Math 4513
MATHEMATICA Assignment 6
due Friday, October 29

This assignment explores the graphs of the real-valued gamma and zeta functions, and the graphs of the 2-variable functions $|\Gamma(x+iy)|$ and $|\zeta(x+iy)|$. These functions are defined in MATHEMATICA as *Gamma[z]* and *Zeta[z]*. As you work on this assignment, you should treat the two different functions in parallel but keep the work for the two separate—so you might create two sections in your notebook (one for gamma and the other for zeta) and subsections for each of the questions below.

1. Sketch plots of the **real-valued** functions $y = \Gamma(x)$ and $y = \zeta(x)$ (where $x \in \mathbb{R}$ with various different windows. Use these plots to estimate the following:
   (a) Those (real) values of x for which $\Gamma(x) = 0$ and $\zeta(x) = 0$.
   (b) The limits of $\Gamma(x)$ and $\zeta(x)$ as x approaches ∞ and $-\infty$.
   (c) The locations of any vertical asymptotes in the graphs $y = \Gamma(x)$ and $y = \zeta(x)$.
   (d) The number (and approximate locations) of the real solutions to the equations $\Gamma(x) = 1$ and $\zeta(x) = 1$.
   (e) The approximate locations of any relative maximums, relative minimums or points of inflection for $y = \Gamma(x)$ and $y = \zeta(x)$. (Suggestion look at the graphs of $y = \Gamma'(x)$, $y = \Gamma''(x)$ $y = \zeta'(x)$ and $y = \zeta''(x)$.)
   (f) Sketch the graphs of $y = |\Gamma(x)|$ and $y = |\zeta(x)|$.

2. Sketch plots of $y = |\Gamma(x + Ci)|$ and $y = |\zeta(x + Ci)|$ for various choices of a real constant C. How do these results fit in with your answers to problem 1?

3. Sketch graphs of the 2-variable functions $z = |\Gamma(x+iy)|$ and $z = |\zeta(x+iy)|$ using the Plot3D command and the ContourPlot command. Explain what you saw in problems 1 and 2 in terms of these pictures.

4. Sketch plots of $x = |\Gamma(C + yi)|$ and $y = |\zeta(C + yi)|$ for various choices of a real constant C (including the critical line where $C = 1/2$). How do these pictures fit in with your answers in problem 3?

5. Sketch graphs of the 2-variable functions $z = 1/|\Gamma(x + iy)|$ and $z = 1/|\zeta(x + iy)|$ using the Plot3D command. Explain how these pictures make it easier to see solutions to the equations $\Gamma(x + iy) = 0$ and $\zeta(x, y) = 0$.