Math 4513
MATHEMATICA Assignment 3
due Monday afternoon Sept. 22

Use MATHEMATICA to solve the following and turn in a print-out with your results. Be sure to include a title cell, as well as comment cells where you state the problems and/or provide discussion of the output.

1. How much of the data in Tables 3-1 and 3-2 (pp. 35 and 39) of Derbyshires's book can you replicate? How did Derbyshire obtain **all** of the data?

2. From problem 2 of the previous assignment, it seems likely that MATHEMATICA uses a routine for determining "PrimePi[k]" which is not based on the sieve method. Root around in the MATHEMATICA help facility (and/or do an internet search) to see if you can learn anything about the routine that MATHEMATICA does use.

3. A prime number p is said to be **Euclidean** if $p = n! + 1$ for some n. Determine the first 16 Euclidean primes and the corresponding values of n.

4. In class we showed that for any positive integer N there exists a prime p for which the open interval $(p, p + N)$ contains no primes. When this happens we say that there is a **gap with length greater or equal to** N in the primes at p.
   Find the first gap in the primes that has length N where $N = 10$. Repeat for $N = 50$.

5. (a) In the first million positive integers what is the largest gap between primes which occurs and where does it occur?
   (b) Write a program that creates a ListPlot of the successive gap sizes between primes in this range (from 2 to 1 million). (In the ListPlot command, include the modifier ",PlotRange–¿All" to make sure that all of the data is shown.)
   (c) What are the gap sizes that occur in this range? (Note: if "list" is a list then "Union[list]" will sort the list and remove any repetitions.)

6. (a) Write a MATHEMATICA routine that computes the number of sets of twin primes between two given integers $M1$ and $M2$. (If there is a gap of length two in the primes at p then $\{p, p + 2\}$ is a **set of twin primes**.)
   (b) Use your answer to (a) to determine the smallest chiliad which contains no twin primes. (A **chiliad** is a block of 1000 consecutive integers between $1000k$ and $1000k + 999$ for some k.)

7. Use MATHEMATICA to sketch a graph of the functions $Li(x)$, $\pi(x)$ and $x/\ln(x)$ similar to the one on page 117 (at the end of Chapter 7) of Derbyshire's book. (Here $Li(x)$ is the **log integral function** denoted by "LogIntegral[x]" in MATHEMATICA.)