MATHEMATICA Assignment 2  
due Wednesday afternoon Sept. 8 
Math 4513

Use MATHEMATICA to solve the following and turn in a print-out with your results. Be sure to include a title cell, as well as comment cells where you state the problems and/or provide discussion of the output.

1. In class we showed that the $M$th partial sum of the harmonic series $H_M = \sum_{n=1}^{M} \frac{1}{n}$ is larger than $\pi^3$ if $M = 2^{63}$. Try to find (or come as close as you can) the smallest value of $M$ with this property (that is, $H_M > \pi^3$ but $H_{M-1} \leq \pi^3$).

2. Here is a MATHEMATICA program that carries out the ‘Sieve of Eratosthenes’ method for finding the primes between 2 and 100.

```
M = 100;
inlist = Range[2, M];
outlist = { }; 
primelist = { }; 
For[k = 1, k <= Sqrt[M], k++,
p = Min[inlist];
primelist = Append[primelist, p];
outlist = Union[outlist, p*Range[1, M/p]]; 
inlist = Complement[inlist, outlist];
]
primelist = Union[primelist, inlist];
```

(a) Use this program to list the primes up to $M$ where $M$ is 100, 1000, and 10000.
(b) Compare the amount of CPU time which the computations in part (a) use with the amount used by invoking the command `Prime[Range[PrimePi[M]]]` for the same values of $M$. How can you account for any time differences?

[NOTE: You might keep track of CPU time used in compiling a cell by adding `start = TimeUsed[]` at the front of the cell, and `Print["Elapsed Time=", TimeUsed[] - start]` at the end.]

3. Get MATHEMATICA to print out all of the primes up to 1000. Try to write your output like Derbyshire does on page 33.

4. (a) Let $M_1$ and $M_2$ be positive integers with $M_1 < M_2$. Write a MATHEMATICA program to determine how many primes there are between $M_1$ and $M_2$ inclusive. (HINT: Use `PrimePi`). Try a few examples such as $M_1 = 3$ and $M_2 = 13$.
(b) Write a MATHEMATICA program to print out all of the primes between $M_1$ and $M_2$ inclusive. Make sure that the `Length` of your answer here agrees with what you found in (a).
(c) Check Derbyshire’s data in the last paragraph on page 33.

5. Use MATHEMATICA to replicate as much of the data as you can in Tables 3-1 and 3-2 (pp. 35 and 39).

6. Enter the following into a MATHEMATICA cell and examine the output:
```
f[A_, B_] = PrimePi[B] - PrimePi[A -.5];
plist = Table[f[N, N + 100], {N, 0, 5000, 100}];
ListPlot[plist, PlotRange -> {0, Max[plist] + 1} ]
```

Explain what the output shows.