Class Problems
Math 4513
November 5, 2004

PROBLEM 1. Consider the subharmonic series $\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} \frac{1}{3^n}$.

a) Show that there are some positive real numbers r which are not the sums of any subseries of $\sum_{n=0}^{\infty} a_n$.

b) Determine all real numbers r with $1 \leq r \leq 2$ which are the sums of (infinite) subseries of $\sum_{n=0}^{\infty} a_n$, or at least find infinitely many such r.

PROBLEM 2. If $\mathcal{N} = \sum_{k=1}^{\infty} \dfrac{1}{n_k}$ is a subharmonic series then let $C(\mathcal{N})$ be the set

$$C(\mathcal{N}) = \{r \in \mathbb{R} \mid r \text{ is the sum of some infinite subseries of } \mathcal{N}\}$$

Given two subharmonic series $\mathcal{N}$ and $\mathcal{M}$ define $\mathcal{N} \leq \mathcal{M}$ to mean that $C(\mathcal{N}) \subset C(\mathcal{M})$. Try to show the following:

a) $\mathcal{N} \leq \mathcal{N}$

b) If $\mathcal{N} \leq \mathcal{M}$ and $\mathcal{M} \leq \mathcal{P}$ then $\mathcal{N} \leq \mathcal{P}$.

c) If $\mathcal{N} \leq \mathcal{M}$ and $\mathcal{M} \leq \mathcal{N}$ then $\mathcal{N} = \mathcal{M}$.

d) Its not always true that either $\mathcal{N} \leq \mathcal{M}$ or $\mathcal{M} \leq \mathcal{N}$.

PROBLEM 3. Examine the MATHEMATICA Experiment shown on the next page.

a) Write some sentences that describe what the MATHEMATICA code is showing.

b) In the displayed graph can you guess which curve represents each of $gap2$, $gap3$, $gap8$ and $gap30$. Why are two of these graphs close while the others are separate?

MATHEMATICA Experiment

```
gap2[x_] :=  (y = 0; For[k = 2, k < PrimePi[x], k++,
      If[PrimeQ[Prime[k] + 2], y = y + 1]];     y)

gap6[x_] :=  (y = 0; For[k = 2, k < PrimePi[x], k++,
      If[PrimeQ[Prime[k] + 6], y = y + 1]];    y)

gap8[x_]:=  (y=0;For[k=2,k<PrimePi[x],k++,
      If[PrimeQ[Prime[k]+8],y=y+1]];          y)

gap30[x_]:=  (y=0;For[k=2,k<PrimePi[x],k++,
      If[PrimeQ[Prime[k]+30],y=y+1]];         y)

Plot[{gap2[x], gap30[x], gap6[x], gap8[x]}, {x, 0, 1000},
  PlotStyle -> {Hue[0.3], Hue[0.8], Hue[.1], Hue[.6]}]
```