Math 4513

Problems involving Arithmetic in the Set of Complex Numbers

9/22/04

Throughout these problems z denotes a complex number with rectangular form $z = x + yi$.

1. Compute $z + \overline{z}$ and $z - \overline{z}$ and use your answer to derive formulas for $Re(z)$ and $Im(z)$ in terms of z and $\overline{z}$.

2. Express $z\overline{z}$ in terms of $|z|$.

3. Show that $\dfrac{1}{1+i} = \dfrac{1}{2}(1-i)$

4. Show that every non-zero complex number $z = x + iy$ has a multiplicative inverse by finding a rectangular form for $\dfrac{1}{x+iy}$ (verify the formula in two different ways).

5. Write $\dfrac{3+2i}{4-3i}$ in rectangular form.

6. Find the rectangular form for each of:

   a) $\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\right)^3$

   b) i^n for any integer n.

   c) $(1+i)^n$ for any integer n.

7. Show that $|zw| = |z||w|$.

8. Use the formula $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ to show that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. (You'll also want to remember the Maclaurin series expansions for $\cos(\theta)$ and $\sin(\theta)$.)

9. Show that $|e^{i\theta}| = 1$ and that every complex number z with $|z| = 1$ equals $e^{i\theta}$ for some θ.

10. Express $\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}$ in polar form. Use this to determine the rectangular form of $\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\right)^n$ for each integer n.