Part I.

1. Let $f$ be the function defined on the interval $(0, \infty)$ given by $f(x) = (3x - 2)/x$. Show that $f$ is continuous.

2. Let $F$ be an ordered field containing a nonzero element $x$. Referring only to the ordered field axioms show that $x^2 > 0$.

3. Let $A \subset \mathbb{R}$. Recall that $x \in \mathbb{R}$ is a limit point for $A$ if and only if every neighborhood of $x$ contains a point of $A$ other than $x$ itself. Show that $x$ is a limit point for $A$ if and only if every neighborhood of $x$ contains infinitely many elements of $A$.

4. Let $f : X \to Y$ be a function. Let $A \subset X$ and $B \subset Y$.
   a) Show that $f(f^{-1}(B)) = B$.
   b) Show that $A \subset f^{-1}(f(A))$.
   c) Give an example where $A \neq f^{-1}(f(A))$.

5. a) State and prove a result describing the limit points of the union $A \cup B$ in terms of the limit points of $A$ and $B$.
   b) Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

Part II. For each problem determine whether the statement is true or false. If true supply a proof. If false provide a counterexample.

5. Let $A \subset \mathbb{R}$. Suppose $\{x_n\}$ is a convergent sequence sequence with $x_n \in A$ for each $n$ and $x = \lim x_n$. Then $x$ is a limit point of $A$.

6. If $f$ is a continuous function with $\mathcal{D}(f) = \mathbb{R}$ and $U \subset \mathbb{R}$ is an open set then $f(U)$ is an open set.

7. Let $A$ be a subset of an ordered field. If $\alpha$ is a lower bound for $A$ which is also an element of $A$ then $\alpha$ is the greatest lower bound for $A$.

8.