Discussion Problems

For each problem determine whether the statement is true or false. If true supply a proof. If false provide a counterexample. All sets are assumed to be subsets of $\mathbb{R}$ and all functions have their domain and range in $\mathbb{R}$.

1. A sequence which is not bounded below cannot diverge to $\infty$.
2. An open interval is compact.
3. If $x$ is an interior point of $A$ then $x$ is not a limit point of $A$.
4. If $x$ is an interior point of $A$ then $x$ is not a boundary point of $A$.
5. A nonempty countable set cannot be open.
6. If $x$ is a boundary point of $A$ then every neighborhood of $x$ contains infinitely many points of $A$ and infinitely many points of $A^c$.
7. If $f$ is a continuous function and $I \subset \mathcal{D}(f)$ is an interval then $f(I)$ is an interval.
8. If $f$ is a continuous function and $A \subset \mathcal{D}(f)$ is bounded then $f(A)$ is bounded.
9. If $f$ is a continuous function and $U \subset \mathcal{D}(f)$ is an open set then $f(U)$ is an open set.
10. A set is open if and only if its complement is closed.
11. Any sequence has at most countably many subsequential limits.
12. Let $A$ be an infinite set which is bounded above. Then the least upper bound of $A$ is a limit point of $A$.
13. The set $\{1/n \mid n \in \mathbb{N}\}$ is closed.
14. The set $\{1/n \mid n \in \mathbb{N}\}$ is connected.
15. Let $f$ be a function whose domain is $\mathbb{R}$. Then $f$ is continuous if and only if the inverse image of every closed set is closed.
16. If $A \subset B$ then $\overline{A} \subset \overline{B}$.
17. The function $f(x) = \sin(1/x)$ is a continuous function.
18. If $f$ is a continuous function whose domain is an interval then its range is also an interval.
19. For any sets $A$ and $B$, $\text{Int}(A \cup B) = \text{Int}(A) \cup \text{Int}(B)$. 