

Proposition: Let A, B and C be sets. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

Proof: Let A, B and C be sets, and let x be an element of A . Since A is a subset of B and $x \in A$, x must be an element of B ² (by the definition of subset)¹. Since $x \in B$ and B is a subset of C , x must be an element of C ² (by the definition of subset)¹. This shows that each element x of A is also an element of C , and so A is a subset of C (by the definition of subset)¹. \square

2: Here's a use of modus ponens.

1: Here's a use of definition of " \subseteq ".